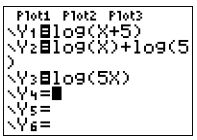
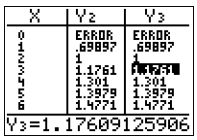
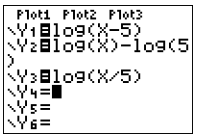
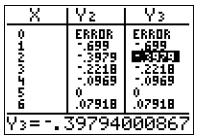




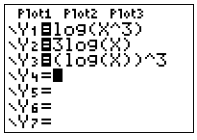
1a $y = \log(x) + \log(5)$ en $y = \log(5x)$ komen op hetzelfde neer.

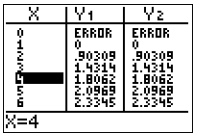
1b $y = \log(x) - \log(5)$ en $y = \log(\frac{x}{5})$ komen op hetzelfde neer.





1c $y = \log(x^3)$ en $y = 3 \cdot \log(x)$ komen op hetzelfde neer.





2a 💻 ${}^2\log(6) + {}^2\log(10) = {}^2\log(6 \cdot 10) = {}^2\log(60)$.

2b 💻 ${}^3\log(30) - {}^3\log(6) = {}^3\log(\frac{30}{6}) = {}^3\log(5)$.

2c 💻 $2 \cdot {}^5\log(3) + {}^5\log(\frac{1}{2}) = {}^5\log(3^2) + {}^5\log(\frac{1}{2}) = {}^5\log(9 \cdot \frac{1}{2}) = {}^5\log(4\frac{1}{2})$.

2d 💻 ${}^{\frac{1}{2}}\log(15) - 4 \cdot {}^{\frac{1}{2}}\log(3) = {}^{\frac{1}{2}}\log(15) - {}^{\frac{1}{2}}\log(3^4) = {}^{\frac{1}{2}}\log(\frac{15}{81}) = {}^{\frac{1}{2}}\log(\frac{5}{27})$.

2e 💻 $-2 \cdot {}^4\log(6) + {}^4\log(12) = {}^4\log(6^{-2}) + {}^4\log(12) = {}^4\log(\frac{1}{36} \cdot 12) = {}^4\log(\frac{12}{36}) = {}^4\log(\frac{1}{3})$.

2f 💻 $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(\frac{50}{25}) = \log(2)$.

3a 💻 $4 + {}^2\log(3) = {}^2\log(2^4) + {}^2\log(3) = {}^2\log(16 \cdot 3) = {}^2\log(48)$.

3b 💻 $3 - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log((\frac{1}{2})^3) - {}^{\frac{1}{2}}\log(10) = {}^{\frac{1}{2}}\log(\frac{1}{8} : 10) = {}^{\frac{1}{2}}\log(\frac{1}{80})$.

3c 💻 $2 - \log(5) = \log(10^2) - \log(5) = \log(\frac{100}{5}) = \log(20)$.

3d 💻 ${}^2\log(12) - {}^3\log(9) = {}^2\log(12) - {}^3\log(3^2) = {}^2\log(12) - 2 = {}^2\log(12) - {}^2\log(2^2) = {}^2\log(\frac{12}{4}) = {}^2\log(3)$.

3e 💻 $\frac{1}{2} \cdot {}^3\log(16) + {}^{\frac{1}{2}}\log(8) = {}^3\log(16^{\frac{1}{2}}) + {}^{\frac{1}{2}}\log(2^3) = {}^3\log(\sqrt{16}) + {}^{\frac{1}{2}}\log((\frac{1}{2})^{-3})$
$= {}^3\log(4) - 3 = {}^3\log(4) - {}^3\log(3^3) = {}^3\log(4) - {}^3\log(27) = {}^3\log(\frac{4}{27})$.

3f 💻 $\log(500) - {}^5\log(125) = \log(500) - {}^5\log(5^3) = \log(500) - 3 = \log(500) - \log(10^3) = \log(\frac{500}{1000}) = \log(\frac{1}{2})$.

4a 💻 ${}^3\log(6) + {}^3\log(1\frac{1}{2}) = {}^3\log(6 \cdot 1\frac{1}{2}) = {}^3\log(9) = {}^3\log(3^2) = 2$.

4b 💻 ${}^5\log(2) - {}^5\log(50) = {}^5\log(\frac{2}{50}) = {}^5\log(\frac{1}{25}) = {}^5\log(5^{-2}) = -2$.

4c 💻 ${}^2\log(27) + 3 \cdot {}^2\log(\frac{1}{6}) = {}^2\log(27) + {}^2\log((\frac{1}{6})^3) = {}^2\log(27 \cdot \frac{1}{216}) = {}^2\log(\frac{27}{216}) = {}^2\log(\frac{1}{8}) = {}^2\log(2^{-3}) = -3$.

4d 💻 $2 \cdot {}^4\log(6) - 2 \cdot {}^4\log(3) = {}^4\log(6^2) - {}^4\log(3^2) = {}^4\log(36) - {}^4\log(9) = {}^4\log(\frac{36}{9}) = {}^4\log(4) = 1$.

5a $g^{{}^g\log(a) - {}^g\log(b)} = \dfrac{g^{{}^g\log(a)}}{g^{{}^g\log(b)}} = \dfrac{a}{b} = g^{{}^g\log(\frac{a}{b})}$, dus ${}^g\log(a) - {}^g\log(b) = {}^g\log(\frac{a}{b})$.

5b $g^{n \cdot {}^g\log(a)} = \left(g^{{}^g\log(a)}\right)^n = a^n = g^{{}^g\log(a^n)}$, dus $n \cdot {}^g\log(a) = {}^g\log(a^n)$.

6a $3 + {}^2\log(3) = {}^2\log(2^3) + {}^2\log(3) = {}^2\log(8 \cdot 3) = {}^2\log(24)$.

6b ${}^2\log(x+1) = 3 + {}^2\log(3)$ BV = beginvoorwaarde: $x + 1 > 0 \Rightarrow x > -1$
${}^2\log(x+1) = {}^2\log(24)$
$x + 1 = 24$
$x = 23$ (voldoet).

7a 💻 ${}^5\log(x) = 3 \cdot {}^5\log(2) - 2 \cdot {}^5\log(3)$ BV: $x > 0$
${}^5\log(x) = {}^5\log(2^3) - {}^5\log(3^2)$
${}^5\log(x) = {}^5\log(\frac{8}{9})$
$x = \frac{8}{9}$ (voldoet).

7b 💻 ${}^2\log(x) = 4 - {}^2\log(3)$ BV: $x > 0$
${}^2\log(x) = {}^2\log(2^4) - {}^2\log(3)$
${}^2\log(x) = {}^2\log(\frac{16}{3})$
$x = \frac{16}{3}$ (voldoet).

7c 💻 ${}^2\log(x+3) = 3 + {}^2\log(x)$ BV: $x > -3$ én $x > 0 \Rightarrow x > 0$
${}^2\log(x+3) = {}^2\log(2^3) + {}^2\log(x)$
${}^2\log(x+3) = {}^2\log(8 \cdot x)$
$x + 3 = 8x$
$-7x = -3$
$x = \frac{3}{7}$ (voldoet).

7d 💻 ${}^3\log(2x) = 1 + {}^3\log(x+1)$ BV: $2x > 0$ én $x > -1 \Rightarrow x > 0$
${}^3\log(2x) = {}^3\log(3) + {}^3\log(x+1)$
${}^3\log(2x) = {}^3\log(3 \cdot (x+1))$
$2x = 3x + 3$
$-x = 3$
$x = -3$ (voldoet niet).

8a $5 \cdot \log(x) = 5 - \log(3125)$ BV: $x > 0$
$\log(x^5) = \log(10^5) - \log(3125)$
$\log(x^5) = \log(\frac{100000}{3125})$
$x^5 = 32 \Rightarrow x = 2$ (voldoet).

100000/3125 32
Ans^(1/5) 2

8b $^{\frac{1}{2}}\log(2x-1) = 2 + {}^{\frac{1}{2}}\log(x+2)$ BV: $x > \frac{1}{2}$
$^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(x+2)$
$^{\frac{1}{2}}\log(2x-1) = {}^{\frac{1}{2}}\log(\frac{1}{4} \cdot (x+2))$
$2x - 1 = \frac{1}{4} \cdot (x+2)$ (links en rechts $\times 4$)
$8x - 4 = x + 2$
$7x = 6 \Rightarrow x = \frac{6}{7}$ (voldoet).

9a $^5\log(x) = 2 + \frac{1}{2} \cdot {}^5\log(3)$ BV: $x > 0$
$^5\log(x) = {}^5\log(5^2) + {}^5\log(3^{\frac{1}{2}})$
$^5\log(x) = {}^5\log(25 \cdot \sqrt{3})$
$x = 25 \cdot \sqrt{3}$ (voldoet).

9b $^3\log(x+4) + 1 = 2 \cdot {}^3\log(x-2)$ BV: $x > 2$
$^3\log(x+4) + {}^3\log(3) = {}^3\log((x-2)^2)$
$^3\log(3 \cdot (x+4)) = {}^3\log((x-2)^2)$
$3x + 12 = x^2 - 4x + 4$
$x^2 - 7x - 8 = 0$
$(x-8) \cdot (x+1) = 0$
$x = 8$ (voldoet) $\vee$ $x = -1$ (vold. niet).

9c $^2\log(2x) - {}^2\log(x+3) = {}^2\log(x) - 2$ BV: $x > 0$
$^2\log(2x) - {}^2\log(x+3) = {}^2\log(x) - {}^2\log(2^2)$
$^2\log(\frac{2x}{x+3}) = {}^2\log(\frac{x}{4})$
$x \cdot (x+3) = 4 \cdot 2x$
$x^2 + 3x = 8x$
$x^2 - 5x = 0$
$x \cdot (x-5) = 0$
$x = 0$ (vold. niet) $\vee$ $x = 5$ (voldoet).

11 $^3\log(4) = \frac{\log(4)}{\log(3)} \approx 1{,}262$ en $^{\frac{1}{2}}\log(3) = \frac{\log(3)}{\log(\frac{1}{2})} \approx -1{,}585$.

log(4)/log(3) 1.261859507
log(3)/log(1/2) -1.584962501

12a $^3\log(3x-5) + {}^{\frac{1}{3}}\log(x-1) = 0$ BV: $x > \frac{5}{3}$
$^3\log(3x-5) - {}^3\log(x-1) = 0$
$^3\log(3x-5) = {}^3\log(x-1)$
$3x - 5 = x - 1$
$2x = 4$
$x = 2$ (voldoet).

12b $^5\log(3x) + 2 \cdot {}^{\frac{1}{5}}\log(x) = 0$ BV: $x > 0$
$^5\log(3x) - {}^5\log(x^2) = 0$
$^5\log(3x) = {}^5\log(x^2)$
$3x = x^2$
$x^2 - 3x = 0$
$x \cdot (x-3) = 0$
$x = 0$ (vold. niet) $\vee$ $x = 3$ (voldoet).

8c $^3\log(x+2) = 1 - {}^3\log(x)$ BV: $x > -2$ én $x > 0 \Rightarrow x > 0$
$^3\log(x+2) = {}^3\log(3) - {}^3\log(x)$
$^3\log(x+2) = {}^3\log(\frac{3}{x})$
$x + 2 = \frac{3}{x}$
$x^2 + 2x = 3$
$x^2 + 2x - 3 = 0$
$(x+3) \cdot (x-1) = 0$
$x = -3$ (vold. niet) $\vee$ $x = 1$ (voldoet).

8d $2 \cdot {}^3\log(x) + 1 = {}^3\log(5x-2)$ BV: $x > 0$ én $x > \frac{2}{5} \Rightarrow x > \frac{2}{5}$
$^3\log(x^2) + {}^3\log(3) = {}^3\log(5x-2)$
$^3\log(3x^2) = {}^3\log(5x-2)$
$3x^2 = 5x - 2$
$3x^2 - 5x + 2 = 0$ (*abc*-formule)
$D = (-5)^2 - 4 \cdot 3 \cdot 2 = 25 - 24 = 1$
$x = \frac{5 \pm \sqrt{1}}{2 \cdot 3} = \frac{5 \pm 1}{6}$
$x = \frac{5+1}{6} = 1$ (voldoet) $\vee$ $x = \frac{5-1}{6} = \frac{2}{3}$ (voldoet).

9d $^3\log(x) = 2 - {}^3\log(x-1)$ BV: $x > 1$
$^3\log(x) = {}^3\log(3^2) - {}^3\log(x-1)$
$^3\log(x) = {}^3\log(\frac{9}{x-1})$
$x \cdot (x-1) = 9$
$x^2 - x - 9 = 0$ (*abc*-formule)
$D = (-1)^2 - 4 \cdot 1 \cdot -9 = 1 + 36 = 37$
$x = \frac{1 \pm \sqrt{37}}{2 \cdot 1} = \frac{1 \pm \sqrt{37}}{2}$
$x = \frac{1-\sqrt{37}}{2} < 0$ (vold. niet) $\vee$ $x = \frac{1+\sqrt{37}}{2} > 1$ (voldoet).

10ab $p^2 - 2p - 8 = 0$
$(p-4)(p+2) = 0$
$p = 4 \vee p = -2$.
$^2\log(x) = 4 \vee {}^2\log(x) = -2$ (BV: $x > 0$)
$x = 2^4 = 16$ (voldoet) $\vee$ $x = 2^{-2} = \frac{1}{4}$ (voldoet).

12c $2x \cdot {}^{\frac{1}{3}}\log(3x+5) = {}^{\frac{1}{3}}\log(3x+5)$ BV: $x > -\frac{5}{3}$
$2x = 1 \vee {}^{\frac{1}{3}}\log(3x+5) = 0$
$x = \frac{1}{2}$ (voldoet) $\vee$ $3x + 5 = (\frac{1}{3})^0 = 1$
$x = \frac{1}{2} \vee 3x = -4$
$x = \frac{1}{2} \vee x = -\frac{4}{3}$ (voldoet).

12d $^2\log^2(x) = 2 \cdot {}^2\log(x) + 3$ BV: $x > 0$
Stel $^2\log(x)$ tijdelijk t
$t^2 = 2t + 3$
$t^2 - 2t - 3 = 0$
$(t-3) \cdot (t+1) = 0$
$t = {}^2\log(x) = 3 \vee t = {}^2\log(x) = -1$
$x = 2^3 = 8$ (voldoet) $\vee$ $x = 2^{-1} = \frac{1}{2}$ (voldoet).

13a $-2\cdot {}^{\frac{1}{2}}\log(x) = 2 + {}^{2}\log(3-x)$ BV: $0<x<3$

$2\cdot {}^{2}\log(x) = {}^{2}\log(2^2) + {}^{2}\log(3-x)$ $\Downarrow$

$x>0$ en $3-x>0$
$x>0$ en $-x>-3$
$x>0$ en $x<3$
$0<x<3$

${}^{2}\log(x^2) = {}^{2}\log(4\cdot(3-x))$

$x^2 = 12-4x$

$x^2+4x-12=0$

$(x+6)\cdot(x-2)=0$

$x=-6$ (vold. niet) $\vee$ $x=2$ (voldoet).

13b ${}^{9}\log(2x) = {}^{3}\log(x-4)$ BV: $x>4$

$\frac{{}^{3}\log(2x)}{{}^{3}\log(3^2)} = {}^{3}\log(x-4)$

${}^{3}\log(2x) = 2\cdot {}^{3}\log(x-4)$

${}^{3}\log(2x) = {}^{3}\log((x-4)^2)$

$2x = x^2-8x+16$

$x^2-10x+16=0$

$(x-8)\cdot(x-2)=0$

$x=8$ (voldoet) $\vee$ $x=2$ (vold. niet).

13c $4x\cdot {}^{4}\log(2x-1) + 3\cdot {}^{4}\log(2x-1) = 0$ BV: $x>\frac{1}{2}$

$(4x+3)\cdot {}^{4}\log(2x-1)=0$

$4x+3=0 \vee {}^{4}\log(2x-1)=0$

$4x=-3 \vee 2x-1=4^0=1$

$x=-\frac{3}{4}$ (vold. niet) $\vee$ $2x=2$

$x=1$ (voldoet).

13d ${}^{\frac{1}{2}}\log^2(x+2) + 3\cdot {}^{\frac{1}{2}}\log(x+2) = 0$ BV: $x>-2$

Stel ${}^{\frac{1}{2}}\log(x+2)$ tijdelijk t

$t^2+3t=0$

$t\cdot(t+3)=0$

$t={}^{\frac{1}{2}}\log(x+2)=0 \vee t={}^{\frac{1}{2}}\log(x+2)=-3$

$x+2=(\frac{1}{2})^0=1 \vee x+2=(\frac{1}{2})^{-3}=(2^{-1})^{-3}=2^3=8$

$x=-1$ (voldoet) $\vee$ $x=6$ (voldoet).

14a $3x\cdot {}^{2}\log(x+1) = {}^{\frac{1}{2}}\log(x+1)$ BV: $x>-1$

$3x\cdot {}^{2}\log(x+1) = -{}^{2}\log(x+1)$

$3x=-1 \vee {}^{2}\log(x+1)=0$

$x=-\frac{1}{3}$ (voldoet) $\vee$ $x+1=2^0=1$

$x=-\frac{1}{3} \vee x=0$ (voldoet).

14b $x^2\cdot {}^{5}\log(2x+1) + 9\cdot {}^{\frac{1}{5}}\log(2x+1) = 0$ BV: $x>-\frac{1}{2}$

$x^2\cdot {}^{5}\log(2x+1) - 9\cdot {}^{5}\log(2x+1) = 0$

$(x^2-9)\cdot {}^{5}\log(2x+1)=0$

$x^2-9=0 \vee 2x+1=5^0=1$

$x^2=9 \vee 2x=0$

$x=3$ (voldoet) $\vee$ $x=-3$ (vold. niet) $\vee$ $x=0$ (voldoet).

14c $2\cdot {}^{3}\log^2(x) + 2 = 5\cdot {}^{3}\log(x)$ BV: $x>0$

Stel ${}^{3}\log(x)$ tijdelijk t

$2t^2+2=5t$

$2t^2-5t+2=0$ (*abc*-formule)

$D=(-5)^2-4\cdot2\cdot2=25-16=9 \Rightarrow t=\frac{5\pm\sqrt{9}}{2\cdot2}=\frac{5\pm3}{4}$

$t={}^{3}\log(x)=\frac{5+3}{4}=\frac{8}{4}=2 \vee t={}^{3}\log(x)=\frac{5-3}{4}=\frac{2}{4}=\frac{1}{2}$

$x=3^2=9$ (voldoet) $\vee$ $x=3^{\frac{1}{2}}=\sqrt{3}$ (voldoet).

14d ${}^{5}\log^2(x) + 3\cdot {}^{\frac{1}{5}}\log(x) + 2 = 0$ BV: $x>0$

${}^{5}\log^2(x) - 3\cdot {}^{5}\log(x) + 2 = 0$

Stel ${}^{5}\log(x)$ tijdelijk t

$t^2-3t+2=0$

$(t-2)\cdot(t-1)=0$

$t={}^{5}\log(x)=2 \vee t={}^{5}\log(x)=1$

$x=5^2=25$ (voldoet) $\vee$ $x=5^1=5$ (voldoet).

15 $2^x=7$ (gegeven) en $2^{{}^{2}\log(7)}=7 \Rightarrow x={}^{2}\log(7)$.

Of: $2^x=7$ (gegeven) $\Rightarrow {}^{2}\log(2^x)={}^{2}\log(7) \Rightarrow x={}^{2}\log(7)$.

Onthoud: $g^{\cdots}$ en ${}^{g}\log\ldots$ heffen elkaar op

16ab $(2^x)^2+2\cdot2^x=8$ (stel $2^x=p$)

$p^2+2p=8$

$p^2+2p-8=0$

$(p+4)\cdot(p-2)=0$

$p=2^x=-4$ (kan niet) $\vee$ $p=2^x=2=2^1 \Rightarrow x=1$.

17a $3^x-2=8\cdot(\frac{1}{3})^x$

$3^x-2=8\cdot\frac{1}{3^x}$ (stel $3^x=t$)

$t-2=\frac{8}{t}$ (links en rechts $\times t$)

$t^2-2t=8$

$t^2-2t-8=0$

$(t-4)\cdot(t+2)=0$

$t=3^x=4 \vee t=3^x=-2$ (kan niet)

$x={}^{3}\log(4)$.

17b $2^x=6-5\cdot(\frac{1}{2})^x$

$2^x=6-5\cdot\frac{1}{2^x}$ (stel $2^x=t$)

$t=6-\frac{5}{t}$ (links en rechts $\times t$)

$t^2=6t-5$

$t^2-6t+5=0$

$(t-5)\cdot(t-1)=0$

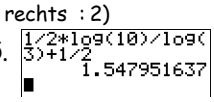
$t=2^x=5 \vee t=2^x=1=2^0$

$x={}^{2}\log(5) \vee x=0$.

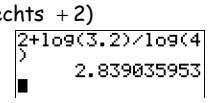
17c $9^x = 4 + 3^{x+1}$
$(3^2)^x = 4 + 3^1 \cdot 3^x$
$(3^x)^2 = 4 + 3 \cdot 3^x$ (stel $3^x = t$)
$t^2 = 4 + 3t$
$t^2 - 3t - 4 = 0$
$(t-4)\cdot(t+1) = 0$
$t = 3^x = 4 \vee t = 3^x = -1$ (kan niet)
$x = {}^3\log(4)$.

17d $2^x = 24 - 2^{2x-1}$
$2^x = 24 - 2^{-1} \cdot (2^x)^2$ (stel $2^x = t$)
$t = 24 - \frac{1}{2}t^2$
$\frac{1}{2}t^2 + t - 24 = 0$ (links en rechts $\times 2$)
$t^2 + 2t - 48 = 0$
$(t+8)\cdot(t-6) = 0$
$t = 2^x = -8$ (kan niet) $\vee$ $t = 2^x = 6$
$x = {}^2\log(6)$.

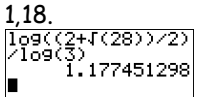
18a $3^{2x-1} = 10$ (links en rechts ${}^3\log\ldots$)
$2x - 1 = {}^3\log(10)$ (links en rechts $+1$)
$2x = {}^3\log(10) + 1$ (links en rechts $:2$)
$x = \frac{1}{2} \cdot {}^3\log(10) + \frac{1}{2} \approx 1{,}55$.



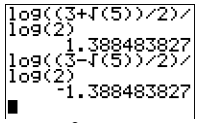
18b $5 \cdot 4^{x-2} = 16$ (links en rechts $:5$)
$4^{x-2} = \frac{16}{5} = \frac{32}{10} = 3{,}2$ (links en rechts ${}^4\log\ldots$)
$x - 2 = {}^4\log(3{,}2)$ (links en rechts $+2$)
$x = 2 + {}^4\log(3{,}2) \approx 2{,}84$.



18c $9^x = 2 \cdot 3^x + 6$
$(3^x)^2 = 2 \cdot 3^x + 6$ (stel $3^x = t$)
$t^2 = 2t + 6$
$t^2 - 2t - 6 = 0$ (*abc*-formule)
$D = (-2)^2 - 4 \cdot 1 \cdot -6 = 4 + 24 = 28 \Rightarrow t = \frac{2 \pm \sqrt{28}}{2 \cdot 1}$
$t = 3^x = \frac{2+\sqrt{28}}{2} \vee t = 3^x = \frac{2-\sqrt{28}}{2} < 0$ (kan niet)
(de tweede teller is neg en de noemer pos $\Rightarrow$ breuk neg)
(links en rechts ${}^3\log\ldots$) $x = {}^3\log(\frac{2+\sqrt{28}}{2}) \approx 1{,}18$.



18d $2^x + 2^{-x} = 3$
$2^x + \frac{1}{2^x} = 3$ (links en rechts $\times 2^x$)
$(2^x)^2 + 1 = 3 \cdot 2^x$ (stel $2^x = t$)
$t^2 + 1 = 3t$
$t^2 - 3t + 1 = 0$ (*abc*-formule)
$D = (-3)^2 - 4 \cdot 1 \cdot 1 = 9 - 4 = 5 \Rightarrow t = \frac{3 \pm \sqrt{5}}{2 \cdot 1}$
$t = 2^x = \frac{3+\sqrt{5}}{2} \vee t = 2^x = \frac{3-\sqrt{5}}{2}$ (links en rechts ${}^2\log\ldots$)
$x = {}^2\log(\frac{3+\sqrt{5}}{2}) \approx 1{,}39 \vee x = {}^2\log(\frac{3-\sqrt{5}}{2}) \approx -1{,}39$.



19a $3^{x+2} + 3^x = 600$
$3^x \cdot 3^2 + 3^x = 600$ (stel $3^x = t$)
$9t + t = 600$
$10t = 600$
$t = 3^x = 60$ (links en rechts ${}^3\log\ldots$)
$x = {}^3\log(60)$.

19b $5^{x-1} + 5^{2x-1} = 4$ (links en rechts $\times 5$)
$5^x + 5^{2x} = 20$ (stel $5^x = t$)
$t + t^2 = 20$
$t^2 + t - 20 = 0$
$(t+5)\cdot(t-4) = 0$
$t = 5^x = -5$ (kan niet) $\vee$ $t = 5^x = 4$ (links en rechts ${}^5\log\ldots$)
$x = {}^5\log(4)$.

19c $3^x + 5 \cdot (\frac{1}{3})^{x-2} = 18$
$3^x + 5 \cdot (\frac{1}{3^x}) \cdot (\frac{1}{3})^{-2} = 18$ (links en rechts $\times 3^x$)
$(3^x)^2 + 5 \cdot 1 \cdot (3^{-1})^{-2} = 18 \cdot 3^x$ (stel $3^x = t$)
$t^2 + 5 \cdot 9 = 18t$
$t^2 - 18t + 45 = 0$
$(t-15)\cdot(t-3) = 0$
$t = 3^x = 15 \vee t = 3^x = 3 = 3^1$
$x = {}^3\log(15) \vee x = 1$.

19d $3^x + 2 \cdot (\frac{1}{3})^{x-2} = 1$
$3^x + 2 \cdot (\frac{1}{3^x}) \cdot (\frac{1}{3})^{-2} = 1$ (links en rechts $\times 3^x$)
$(3^x)^2 + 2 \cdot 1 \cdot 9 = 1 \cdot 3^x$ (stel $3^x = t$)
$t^2 + 18 = t$
$t^2 - t + 18 = 0$ (*abc*-formule)
$D = (-1)^2 - 4 \cdot 1 \cdot 18 < 0 \Rightarrow$ geen oplossing.

20a $y = 2^x \xrightarrow{\text{translatie } (-3,\,0)} f(x) = 2^{x+3}$.

20b $y = 2^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 8} f(x) = 8 \cdot 2^x = 2^3 \cdot 2^x = 2^{x+3}$.

21a $y = {}^2\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{8}} f(x) = {}^2\log(8 \cdot x)$.

21b $y = {}^2\log(x) \xrightarrow{\text{translatie } (0,\,3)} f(x) = {}^2\log(x) + 3 = {}^2\log(x) + {}^2\log(2^3) = {}^2\log(x) + {}^2\log(8) = {}^2\log(8 \cdot x)$.

22a $y = 2^x \xrightarrow{\text{translatie } (5,\,0)} f(x) = 2^{x-5} = 2^x \cdot 2^{-5} = \frac{1}{2^5} \cdot 2^x = \frac{1}{32} \cdot 2^x \xleftarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{32}} y = 2^x$.

22b $y = 4^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 2} f(x) = 2 \cdot 4^x = \sqrt{4} \cdot 4^x = 4^{\frac{1}{2}} \cdot 4^x = 4^{x+\frac{1}{2}} \xleftarrow{\text{translatie } (-\frac{1}{2},\,0)} y = 4^x$.

22c $y = {}^2\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{32}} f(x) = {}^2\log(32 \cdot x) = {}^2\log(x) + {}^2\log(32) = {}^2\log(x) + {}^2\log(2^5) = {}^2\log(x) + 5$.
$y = {}^2\log(x) \xrightarrow{\text{translatie } (0,\,5)} f(x) = {}^2\log(x) + 5$.

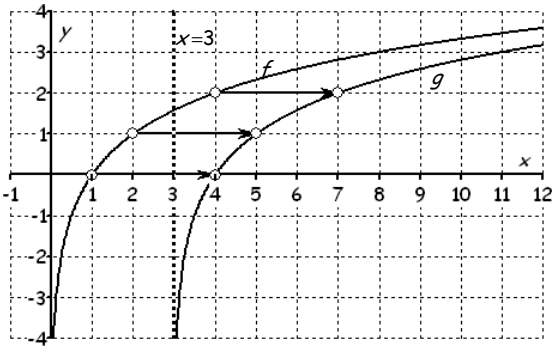
22d $y = {}^4\log(x) \xrightarrow{\text{translatie } (0, \frac{1}{2})} f(x) = {}^4\log(x) + \frac{1}{2} = {}^4\log(x) + {}^4\log(4^{\frac{1}{2}}) = {}^4\log(x \cdot 4^{\frac{1}{2}}) = {}^4\log(x \cdot \sqrt{4}) = {}^4\log(2 \cdot x)$.

$y = {}^4\log(x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{2}} f(x) = {}^4\log(2 \cdot x)$.

23a $f(x) = {}^2\log(x) \xrightarrow{\text{translatie } (3, 0)} g(x) = {}^2\log(x - 3)$.

23b Zie de grafiek hiernaast.

23c Nee. (je kunt de vert. asymptoot $x = 0$ niet op $x = 3$ krijgen met een vermenigvuldiging t.o.v. x-as of een vermenigvuldiging t.o.v. y-as)



23d $g(x) = {}^2\log(x - 3) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } \frac{1}{4}} h(x) = {}^2\log(4x - 3)$.
$h(x) = {}^2\log(4x - 3) = {}^2\log(4 \cdot (x - \frac{3}{4})) = {}^2\log(4) + {}^2\log(x - \frac{3}{4})$.
Dus $p = -\frac{3}{4}$ en $q = {}^2\log(4) = {}^2\log(2^2) = 2$.

24a $f(x) = (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } 4} g(x) = 4 \cdot (\frac{1}{2})^x$.

24b Er geldt: $4 \cdot (\frac{1}{2})^x = 2^2 \cdot (\frac{1}{2})^x = (\frac{1}{2})^{-2} \cdot (\frac{1}{2})^x = (\frac{1}{2})^{x-2}$. Dus $f(x) = (\frac{1}{2})^x \xrightarrow{\text{translatie } (2, 0)} g(x) = (\frac{1}{2})^{x-2}$.

24cd $g(x) = 4 \cdot (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{4}} f(x) = (\frac{1}{2})^x \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } -\frac{1}{2}} h(x) = (\frac{1}{2})^{-2x} = \left((2^{-1})^{-2}\right)^x = 4^x$.

24e $g(x) = 4 \cdot (\frac{1}{2})^x \xrightarrow{\text{translatie } (3, 4)} j(x) = 4 \cdot (\frac{1}{2})^{x-3} + 4$.
$j(x) = 4 \cdot (\frac{1}{2})^{x-3} + 4 = 4 \cdot (\frac{1}{2})^x \cdot (\frac{1}{2})^{-3} + 4 = 4 \cdot (\frac{1}{2})^x \cdot 2^3 + 4 = 32 \cdot (\frac{1}{2})^x + 4$. Dus $a = 32$ en $b = 4$.

25 $|AB| = g(1) - f(1) = 8 - 2^1 - 2^{1-2} = 8 - 2 - 2^{-1} = 6 - \frac{1}{2} = 5\frac{1}{2}$.

💻

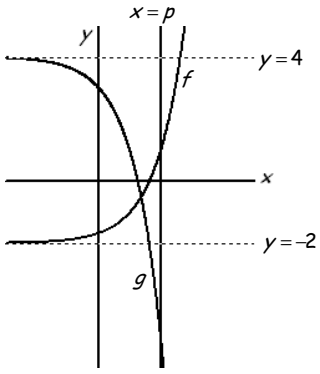
26a $x = {}^2\log(6\frac{2}{5}) \Rightarrow y_S = g({}^2\log(6\frac{2}{5})) = 8 - 2^{{}^2\log(6\frac{2}{5})} = 8 - 6\frac{2}{5} = 1\frac{3}{5}$.

26b Links van het snijpunt S is de afstand altijd kleiner dan 8, omdat de lijnen $y = 0$ en $y = 8$ asymptoten zijn van respectievelijk de grafieken van f en g. Rechts van het snijpunt S kan de afstand wel gelijk worden aan 10. Dus er precies één waarde van p waarvoor de grafieken een lijnstuk met lengte 10 afsnijden van de lijn $x = p$.

26c Er zijn twee waarden van p voor $0 < a < 8$.

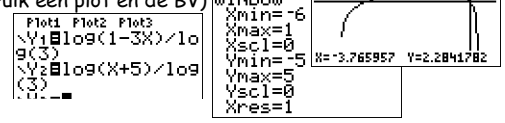
27a $f(x) = g(x)$ geeft:
$3^{x-1} - 2 = 4 - 3^x$
$3^x \cdot 3^{-1} - 2 = 4 - 3^x$
$\frac{1}{3} \cdot 3^x - 2 = 4 - 3^x$
$1\frac{1}{3} \cdot 3^x = 6$
$3^x = 6 \cdot \frac{3}{4} = \frac{18}{4} = \frac{9}{2} = 4\frac{1}{2}^{\bullet\bullet\bullet}$
$x_A = {}^3\log(4\frac{1}{2})$
$y_A = g({}^3\log(4\frac{1}{2})) = 4 - 4\frac{1}{2}^{\bullet\bullet\bullet} = -\frac{1}{2}$.

27b $f(p) - g(p) = 6$
(kan alleen rechts van A)
$3^{p-1} - 2 - (4 - 3^p) = 6$
$3^p \cdot 3^{-1} - 2 - 4 + 3^p = 6$
$\frac{1}{3} \cdot 3^p - 6 + 3^p = 6$
$1\frac{1}{3} \cdot 3^p = 12$
$3^p = 12 \cdot \frac{3}{4} = 9 = 3^2$
$p = 2$.



28a $f(x) = {}^3\log(1 - 3x)$ (BV: $1 - 3x > 0 \Rightarrow -3x > -1 \Rightarrow x < \frac{1}{3} \Rightarrow$) met $D_f = \langle \leftarrow, \frac{1}{3} \rangle$.
$g(x) = {}^3\log(x + 5)$ (BV: $x + 5 > 0 \Rightarrow x > -5 \Rightarrow$) met $D_g = \langle -5, \rightarrow \rangle$.

28b $f(x) = g(x)$ geeft:
${}^3\log(1 - 3x) = {}^3\log(x + 5)$ BV: $-5 < x < \frac{1}{3}$
$1 - 3x = x + 5$
$-4x = 4$
$x_S = -1$ (voldoet).
$f(x) \leq g(x) \Rightarrow -1 \leq x < \frac{1}{3}$.
(gebruik een plot en de BV)



28c Links van $S \Rightarrow -5 < p < -1$ ∨ rechts van $S \Rightarrow -1 < p < \frac{1}{3}$
$f(p) - g(p) = 2$ BV: $-5 < p < \frac{1}{3}$ ∨ $g(p) - f(p) = 2$ BV: $-5 < p < \frac{1}{3}$
${}^3\log(1 - 3p) - {}^3\log(p + 5) = 2$ ∨ ${}^3\log(p + 5) - {}^3\log(1 - 3p) = 2$
${}^3\log(\frac{1 - 3p}{p + 5}) = {}^3\log(3^2)$ ∨ ${}^3\log(\frac{p + 5}{1 - 3p}) = {}^3\log(3^2)$
$\frac{1 - 3p}{p + 5} = 9$ ∨ $\frac{p + 5}{1 - 3p} = 9$
$1 - 3p = 9p + 45$ ∨ $p + 5 = 9 - 27p$
$-12p = 44$ ∨ $28p = 4$
$p = -\frac{44}{12} = -\frac{11}{3} = -3\frac{2}{3}$ (voldoet). ∨ $p = \frac{4}{28} = \frac{1}{7}$ (voldoet).

29a $f(x) = g(x)$ geeft:

$(\frac{3}{2})^{x+2} = 3 \cdot (\frac{2}{3})^x + 3$

$(\frac{3}{2})^x \cdot (\frac{3}{2})^2 = 3 \cdot (\frac{3}{2})^{-x} + 3$

$\frac{9}{4} \cdot (\frac{3}{2})^x = 3 \cdot (\frac{3}{2})^{-x} + 3$ (stel $(\frac{3}{2})^x = t$)

$\frac{9}{4}t = 3 \cdot \frac{1}{t} + 3$ (links en rechts $\times t$)

$\frac{9}{4}t^2 = 3 \cdot 1 + 3t$ (links en rechts $\times 4$)

$9t^2 = 12 + 12t$ (links en rechts : 3)

$3t^2 = 4 + 4t$

$3t^2 - 4t - 4 = 0$ (*abc*-formule)

$D = (-4)^2 - 4 \cdot 3 \cdot -4 = 16 + 48 = 64$, dus $\sqrt{D} = 8$

$t = (\frac{3}{2})^x = \frac{4+8}{2 \cdot 3} = 2$ $\vee$ $t = (\frac{3}{2})^x = \frac{4-8}{6} < 0$ (kan niet)

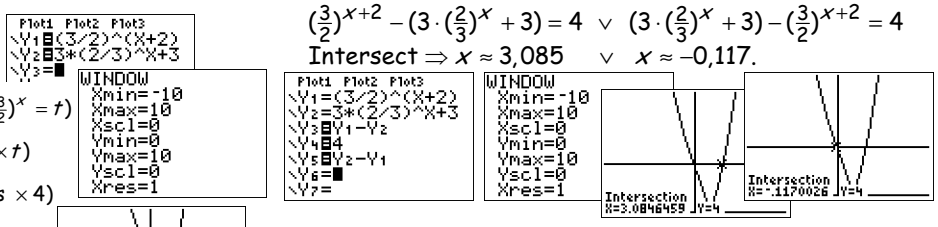
$x_S = {}^{\frac{3}{2}}\log(2)$ en $y_S = g({}^{\frac{3}{2}}\log(2)) = 3 \cdot (\frac{3}{2})^{-{}^{\frac{3}{2}}\log(2)} + 3$

$= 3 \cdot (\frac{3}{2})^{{}^{\frac{3}{2}}\log(2^{-1})} + 3 = 3 \cdot \frac{1}{2} + 3 = 4\frac{1}{2}$.

29b $f(p) - g(p) = 4$ $\vee$ $g(p) - f(p) = 4$

$(\frac{3}{2})^{x+2} - (3 \cdot (\frac{2}{3})^x + 3) = 4$ $\vee$ $(3 \cdot (\frac{2}{3})^x + 3) - (\frac{3}{2})^{x+2} = 4$

Intersect $\Rightarrow x \approx 3{,}085$ $\vee$ $x \approx -0{,}117$.



30a $x_B = x_A + AB = x_A + 6 = p + 6$.

30b $\left.\begin{array}{l} y_A = f(p) \\ y_B = g(p+6) \\ y_A = y_B \end{array}\right\} \Rightarrow f(p) = g(p+6)$.

$f(p) = g(p+6) \Rightarrow {}^2\log(4p) = {}^2\log(p+6)$ (BV: $p > 0$)

$4p = p + 6 \Rightarrow 3p = 6 \Rightarrow p = 2$ (voldoet).

30c $q = y_A = f(p) = f(2) = {}^2\log(8) = {}^2\log(2^3) = 3$.

31 Boven S: $g(p) = f(p + 1\frac{1}{8}) = q$

$2 + {}^{\frac{1}{2}}\log(p+2) = {}^{\frac{1}{2}}\log(2(p+1\frac{1}{8}))$

$^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(p+2) = {}^{\frac{1}{2}}\log(2p + 2\frac{1}{4})$

$^{\frac{1}{2}}\log(\frac{1}{4} \cdot (p+2)) = {}^{\frac{1}{2}}\log(2p + 2\frac{1}{4})$

$\frac{1}{4} \cdot (p+2) = 2p + 2\frac{1}{4}$

$p + 2 = 8p + 9$

$-7p = 7$

$p = -1 \Rightarrow q = g(p) = 2 + {}^{\frac{1}{2}}\log(1) = 2 + {}^{\frac{1}{2}}\log((\frac{1}{2})^0) = 2 + 0 = 2$.

Onder S: $f(p) = g(p + 1\frac{1}{8}) = q$

$^{\frac{1}{2}}\log(2p) = 2 + {}^{\frac{1}{2}}\log(p + 1\frac{1}{8} + 2)$

$^{\frac{1}{2}}\log(2p) = {}^{\frac{1}{2}}\log((\frac{1}{2})^2) + {}^{\frac{1}{2}}\log(p + 3\frac{1}{8})$

$^{\frac{1}{2}}\log(2p) = {}^{\frac{1}{2}}\log(\frac{1}{4} \cdot (p + 3\frac{1}{8}))$

$2p = \frac{1}{4} \cdot (p + 3\frac{1}{8})$

$8p = p + 3\frac{1}{8}$

$7p = 3\frac{1}{8} = \frac{25}{8}$

$p = \frac{25}{56} \Rightarrow q = f(p) = {}^{\frac{1}{2}}\log(\frac{50}{56}) = {}^{\frac{1}{2}}\log(\frac{25}{28})$.

32a Omdat $x = -2$ en $x = 0$ de asymptoten van de grafieken van f en g zijn, nadert de lengte van het lijnstuk dat de grafieken van de lijn $y = q$ (boven het snijpunt) afsnijden tot 2 als q steeds groter wordt.
Alleen van een lijn $y = q$ die onder het snijpunt ligt kunnen de grafieken een lijnstuk met lengte 3 afsnijden.

32b Voor $0 < a < 2$.

33 Onder S: $f(p) = g(p+2) = q$

$2^{p-2} = 8 - 2^{p+2}$

$2^p \cdot 2^{-2} = 8 - 2^p \cdot 2^2$

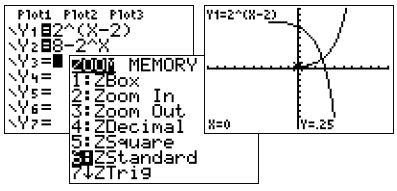
$\frac{1}{4} \cdot 2^p = 8 - 4 \cdot 2^p$

$4\frac{1}{4} \cdot 2^p = 8$

$17 \cdot 2^p = 32$

$2^p = \frac{32}{17}$ *

$q = f(p) = \frac{1}{4} \cdot \frac{32}{17}$ * $= \frac{8}{17}$.



Boven S: $g(p) = f(p+2) = q$

$8 - 2^p = 2^{p-2+2}$

$8 - 2^p = 2^p$

$8 = 2 \cdot 2^p$

$2^p = 4$ *

$q = g(p) = 8 - 4$* $= 4$.

34a ${}^4\log(x^2 - 1) = {}^2\log(x+3)$ BV: $|x| > 1$ en $x > -3 \Rightarrow -3 < x < -1$ of $x > 1$

$\frac{{}^2\log(x^2-1)}{{}^2\log(4)} = {}^2\log(x+3)$ (met ${}^2\log(4) = {}^2\log(2^2) = 2$)

${}^2\log(x^2 - 1) = 2 \cdot {}^2\log(x+3)$

${}^2\log(x^2 - 1) = {}^2\log((x+3)^2)$

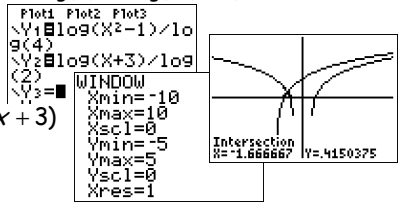
$x^2 - 1 = (x+3)^2 = (x+3) \cdot (x+3)$

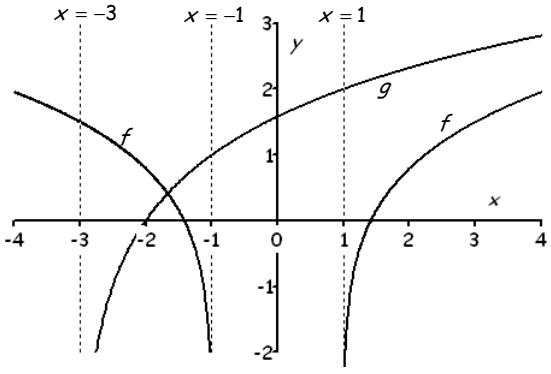
$x^2 - 1 = x^2 + 6x + 9$

$-6x = 10$

$x = -\frac{10}{6} = -\frac{5}{3}$ (voldoet).

$f(x) \le g(x)$ (gebruik een grafiek en de BV) $\Rightarrow -\frac{5}{3} \le x < -1 \vee x > 1$.





34b Links van S: (als f boven g loopt $\Rightarrow -3 < x < -\frac{5}{3}$)

$f(p) - g(p) = \frac{1}{2}$

$^4\log(p^2-1) - {}^2\log(p+3) = \frac{1}{2}$

$\frac{^2\log(p^2-1)}{^2\log(4)} - {}^2\log(p+3) = \frac{1}{2}$

$\frac{^2\log(p^2-1)}{2} - {}^2\log(p+3) = \frac{1}{2}$ (links en rechts $\times 2$)

$^2\log(p^2-1) - 2\cdot{}^2\log(p+3) = 1$

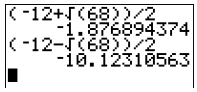
$^2\log(p^2-1) - {}^2\log((p+3)^2) = {}^2\log(2)$

$\frac{p^2-1}{(p+3)^2} = 2$

$p^2 - 1 = 2(p^2+6p+9)$

$p^2 - 1 = 2p^2 + 12p + 18$

$p^2 + 12p + 19 = 0$ (*abc*-formule)



$D = 12^2 - 4\cdot 1\cdot 19 = 144 - 76 = 68 \Rightarrow \sqrt{D} = \sqrt{68} = 2\cdot\sqrt{17}$

$p = \frac{-12+2\sqrt{17}}{2}$ (voldoet) $\vee$ $p = \frac{-12-2\sqrt{17}}{2}$ (vold. niet).

Rechts van S: (als g boven $f \Rightarrow -\frac{5}{3} < x < -1$ of $x > 1$)

$g(p) - f(p) = \frac{1}{2}$

$^2\log(p+3) - {}^4\log(p^2-1) = \frac{1}{2}$

$^2\log(p+3) - \frac{^2\log(p^2-1)}{2} = \frac{1}{2}$ (links en rechts $\times 2$)

$2\cdot{}^2\log(p+3) - {}^2\log(p^2-1) = 1$

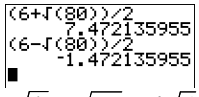
$^2\log((p+3)^2) - {}^2\log(p^2-1) = {}^2\log(2)$

$\frac{(p+3)^2}{p^2-1} = 2$

$2\cdot(p^2-1) = p^2 + 6p + 9$

$2p^2 - 2 = p^2 + 6p + 9$

$p^2 - 6p - 11 = 0$ (*abc*-formule)



$D = (-6)^2 - 4\cdot 1\cdot -11 = 36 + 44 = 80 \Rightarrow \sqrt{D} = \sqrt{80} = 4\sqrt{5}$

$p = \frac{6+4\sqrt{5}}{2}$ (voldoet) $\vee$ $p = \frac{6-4\sqrt{5}}{2}$ (voldoet).

Dus $p = -6 + \sqrt{17} \vee p = 3 + 2\sqrt{5} \vee p = 3 - 2\sqrt{5}$.

34c Boven S: $f(p) = g(p+1) = q$ (met $p < -\frac{5}{3}$)

$^4\log(p^2-1) = {}^2\log(p+1+3)$

$\frac{1}{2}\cdot{}^2\log(p^2-1) = {}^2\log(p+4)$

$^2\log(p^2-1) = 2\cdot{}^2\log(p+4)$

$^2\log(p^2-1) = {}^2\log((p+4)^2)$

$p^2 - 1 = p^2 + 8p + 16$

$-17 = 8p$

$p = -\frac{17}{8}$ (voldoet).

Onder S: $g(p) = f(p+1) = q$ (met $p < -\frac{5}{3}$)

$^2\log(p+3) = {}^4\log((p+1)^2 - 1)$

$^2\log(p+3) = \frac{1}{2}\cdot{}^2\log(p^2+2p)$

$2\cdot{}^2\log(p+3) = {}^2\log(p^2+2p)$

$^2\log((p+3)^2) = {}^2\log(p^2+2p)$

$p^2 + 6p + 9 = p^2 + 2p$

$4p = -9$

$p = -\frac{9}{4}$ (voldoet).

$p = -2\frac{1}{8} \Rightarrow q = f(p) = g(p+1) = g(-1\frac{1}{8}) = {}^2\log(-1\frac{1}{8}+3) = {}^2\log(1\frac{7}{8}) = {}^2\log(\frac{15}{8}) = {}^2\log(15) - {}^2\log(8) = {}^2\log(15) - 3.$

$p = -\frac{9}{4} \Rightarrow q = g(p) = g(-\frac{9}{4}) = {}^2\log(-\frac{9}{4}+3) = {}^2\log(\frac{3}{4}) = {}^2\log(3) - {}^2\log(4) = {}^2\log(3) - 2.$

35a $\left.\begin{array}{l} x_B = p \Rightarrow AB = p \\ AB : BC = 1:2 \Rightarrow BC = 2p \end{array}\right\} \Rightarrow AC = AB + BC = 3p \Rightarrow x_C = 3p.$

35b $f(x_B) = g(x_C) = q$ omdat B en C op de lijn $y = q$ liggen.

Dus $f(p) = g(3p)$ en hieruit volgt $2^p = 2^{3p-3}$.

$2^p = 2^{3p-3}$

$p = 3p - 3$

$-2p = -3$

$p = \frac{3}{2} = 1\frac{1}{2}$.

35c $q = f(x_B) = f(p) = f(1\frac{1}{2}) = 2^{1\frac{1}{2}} = 2^1 \cdot 2^{\frac{1}{2}} = 2\cdot\sqrt{2}.$

36 Stel $x_B = p$ dan $x_C = 2p$.

$f(p) = f(2p) = q$ geeft:

$6p\cdot 2^{-p} = 6\cdot 2p\cdot 2^{-2p}$ (links en rechts $\times 2^{2p}$)

$6p\cdot 2^p = 12p$

$6p = 0 \vee 2^p = 2 = 2^1$

$p = 0$ (vold. niet) $\vee$ $p = 1$.

37a Stel $x_B = p$ dan $x_C = 3p$.

$f(p) = g(3p) = q$ geeft:

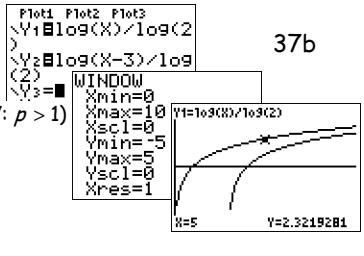
$^2\log(p) = {}^2\log(3p-3)$ (BV: $p > 1$)

$p = 3p - 3$

$-2p = -3$

$p = \frac{3}{2}$ (voldoet).

$q = f(p) = f(\frac{3}{2}) = {}^2\log(\frac{3}{2})$.

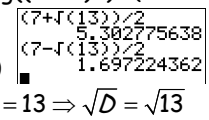


37b $y_F = 2\cdot y_E \Rightarrow f(r) = 2\cdot g(r)$.

$^2\log(r) = 2\cdot{}^2\log(r-3) = {}^2\log((r-3)^2)$ (BV: $r > 3$)

$r = r^2 - 6r + 9$

$r^2 - 7r + 9 = 0$ (*abc*-formule)



$D = (-7)^2 - 4\cdot 1\cdot 9 = 49 - 36 = 13 \Rightarrow \sqrt{D} = \sqrt{13}$

$r = \frac{7+\sqrt{13}}{2} \approx 5{,}303$ (voldoet) $\vee$ $r = \frac{7-\sqrt{13}}{2}$ (vold. niet).

38 Stel $x_B = p$ dan $x_C = 3p$.

$f(p) = f(3p) = q$

$8p\cdot(\frac{1}{3})^p = 8\cdot 3p\cdot(\frac{1}{3})^{3p}$

$p = 0$ (vold. niet) $\vee$ $(\frac{1}{3})^p = 3\cdot(\frac{1}{3})^{3p}$

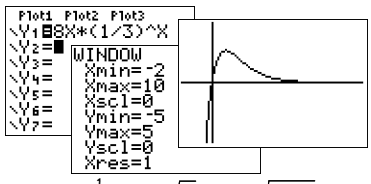
$(\frac{1}{3})^p = (\frac{1}{3})^{-1}\cdot(\frac{1}{3})^{3p}$

$(\frac{1}{3})^p = (\frac{1}{3})^{3p-1}$

$p = 3p - 1$

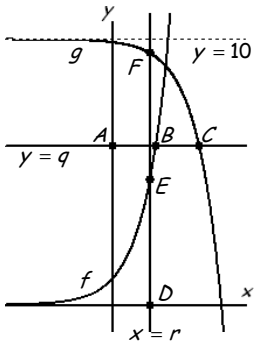
$-2p = -1$

$p = \frac{1}{2}$



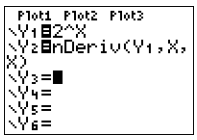
$q = f(p) = f(\frac{1}{2}) = 8\cdot\frac{1}{2}\cdot(\frac{1}{3})^{\frac{1}{2}} = 4\cdot\sqrt{\frac{1}{3}} = 4\cdot\sqrt{\frac{1}{3}\cdot\frac{3}{3}} = \frac{4}{3}\cdot\sqrt{3}.$

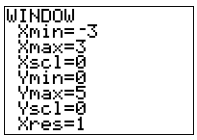
39a Stel $x_B = p$ dan $x_C = 2p$.
$f(p) = g(2p) = q$
$3^p = 10 - 3^{2p-2}$ (links en rechts $\times 3^2$)
$9 \cdot 3^p = 90 - 3^{2p}$
$3^{2p} + 9 \cdot 3^p - 90 = 0$ (stel $3^p = t$)
$t^2 + 9t - 90 = 0$
$(t+15)(t-6) = 0$
$t = 3^p = -15$ (kan niet) $\vee$ $t = 3^p = 6$.
Dus $q = f(p) = 3^p = 6$.

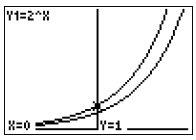


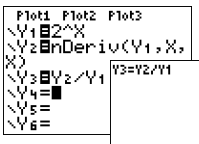
39b $y_F = 2 \cdot y_E \Rightarrow g(r) = 2 \cdot f(r)$.
$10 - 3^{r-2} = 2 \cdot 3^r$ (links en rechts $\times 3^2$)
$90 - 3^r = 18 \cdot 3^r$
$-19 \cdot 3^r = -90$
$3^r = \frac{90}{19}$
$r = {}^3\log(\frac{90}{19})$.

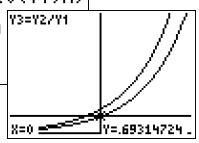
40a

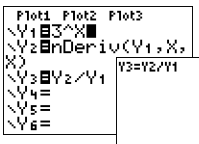


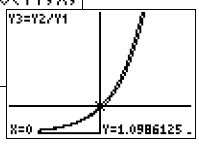












40b $y = 2^x \Rightarrow y' = ????$, maar $\frac{y'}{y} = c \approx 0{,}6931 \Rightarrow y' = c \cdot y = c \cdot 2^x$.

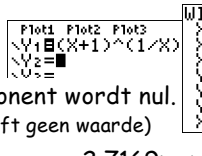
40c $y = 3^x \Rightarrow y' = ????$, maar $\frac{y'}{y} = c \approx 1{,}0986 \Rightarrow y' = c \cdot y = c \cdot 3^x$.

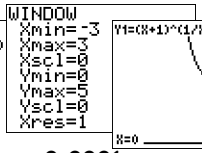
41a $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \frac{2^{x+h} - 2^x}{h} = \lim\limits_{h \to 0} \frac{2^x \cdot 2^h - 2^x}{h} = \lim\limits_{h \to 0} \frac{2^x \cdot (2^h - 1)}{h} = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$.

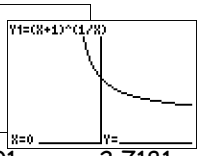
41b $f'(0) = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^0$ (zie 41a) $= \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 1 = \lim\limits_{h \to 0} \frac{2^h - 1}{h}$.

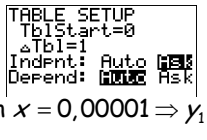
41c $f(x) = 2^x \Rightarrow f'(x) = \lim\limits_{h \to 0} \frac{2^h - 1}{h} \cdot 2^x$ (zie 41a) $= f'(0)$ (zie 41b) $\cdot 2^x$.

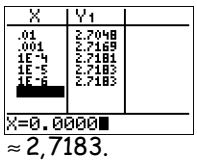
42a Zie de plot hiernaast.











42b De noemer van de breuk in de exponent wordt nul.
(een breuk waarvan de noemer nul is heeft geen waarde)

42c $x = 0{,}01 \Rightarrow y_1 \approx 2{,}7048$; $x = 0{,}001 \Rightarrow y_1 \approx 2{,}7169$; $x = 0{,}0001 \Rightarrow y_1 \approx 2{,}7181$ en $x = 0{,}00001 \Rightarrow y_1 \approx 2{,}7183$.

42d Voor het getal $a \approx 2{,}718$ geldt: $f(x) = a^x \Rightarrow f'(x) = a^x$.

43a 💻 $2e^2 - e^2 = 2e^2 - 1e^2 = 1e^2 = e^2$.

43b 💻 $4\sqrt{e} - \sqrt{e} = 4\sqrt{e} - 1\sqrt{e} = 3\sqrt{e}$.

43c 💻 $5e^2 \cdot 3e^3 = 15e^{2+3} = 15e^5$.

43d 💻 $\frac{12e^6}{4e^2} = 3e^{6-2} = 3e^4$.

43e 💻 $e^{5x} \cdot e^x = e^{5x+x} = e^{6x}$.

43f 💻 $e^x \cdot e^2 = e^{x+2}$.

43g 💻 $5e^x - 3e^x = 2e^x$.

43h 💻 $e^x \cdot (e^2 + 1) = e^{x+2} + e^x$.

43i 💻 $e^x \cdot (e^x + 1) = e^{2x} + e^x$.

43j 💻 $(e^x + 1)^2 = e^{2x} + 2e^x + 1$.

43k 💻 $(e^{3x} + 3)^2 = e^{6x} + 6e^{3x} + 9$.

43l 💻 $\frac{6e^{2x} - e^x}{e^x} = 6e^x - 1$.

44a $(2 + 3e^{\frac{1}{2}x})^2 = 2^2 + 2 \cdot 2 \cdot 3e^{\frac{1}{2}x} + (3e^{\frac{1}{2}x})^2 = 4 + 12e^{\frac{1}{2}x} + 9e^x$. Gebruik: $(\square + \Delta)^2 = (\square + \Delta) \cdot (\square + \Delta) = \square^2 + 2\square\Delta + \Delta^2$.

44b $(e^x + e^{-x})^2 = (e^x)^2 + 2 \cdot e^x \cdot e^{-x} + (e^{-x})^2 = e^{2x} + 2 + e^{-2x}$. ($e^0 = 1$)

44c $\frac{e^{2x} - 4}{e^x - 2} = \frac{(e^x + 2) \cdot (e^x - 2)}{e^x - 2} = e^x + 2$ ($e^x - 2 \neq 0 \Rightarrow e^x \neq 2$) Gebruik: $(\square + \Delta) \cdot (\square - \Delta) = \square^2 - \square\Delta + \square\Delta - \Delta^2 = \square^2 - \Delta^2$.

45a 💻 $(2x + 4) \cdot e^x = 0$
$2x + 4 = 0 \vee e^x = 0$ (kan niet)
$2x = -4$
$x = -2$.

45b 💻 $x^2 \cdot e^x = 3x \cdot e^x$
$e^x = 0$ (kan niet) $\vee x^2 = 3x$
$x = 0 \vee x = 3$.

45c 💻 $x^2 \cdot e^x = 1 \cdot e^x$
$e^x = 0$ (kan niet) $\vee x^2 = 1$
$x = \pm 1$
$x = -1 \vee x = 1$.

45d 💻 $e^{3x} - e^x = 0$
$e^{3x} = e^x$
$3x = x$
$x = 0$.

45e 💻 $e^{4x} - 1 = 0$
$e^{4x} = 1 = e^0$
$4x = 0$
$x = 0$.

45f 💻 $e^x \cdot e^x = e^6$
$e^{2x} = e^6$
$2x = 6$
$x = 3$.

46a $e^x + e^x = 2e^6$
$2e^x = 2e^6$
$e^x = e^6$
$x = 6$.

46b $\frac{e^{5x}}{e^x} = e$
$e^{4x} = e^1$
$4x = 1$
$x = \frac{1}{4}$.

46c $2xe^x + e^x = 0$
$e^x \cdot (2x+1) = 0$
$e^x = 0$ (kan niet) $\vee\ 2x = -1$
$x = -\frac{1}{2}$.

46d $e^{x+2} - \sqrt{e} = 0$
$e^{x+2} = e^{\frac{1}{2}}$
$x + 2 = \frac{1}{2}$
$x = -1\frac{1}{2}$.

46e $e^{2x} + e^x = 2$ (stel $e^x = t$)
$t^2 + t - 2 = 0$
$(t+2)(t-1) = 0$
$t = e^x = -2$ (kan niet) $\vee\ t = e^x = 1 = e^0$
$x = 0$.

46f $e^{6x} + 1 = 2e^{3x}$ (stel $e^{3x} = t$)
$t^2 - 2t + 1 = 0$
$(t-1)(t-1) = 0$
$t = e^{3x} = 1 = e^0$
$3x = 0 \Rightarrow x = 0$.

47a $f(x) = x \cdot e^x \Rightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x \cdot (1+x)$.

47b $f(x) = \frac{e^x}{x+1} \Rightarrow f'(x) = \frac{(x+1)\cdot e^x - e^x \cdot 1}{(x+1)^2} = \frac{xe^x + e^x - e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}$.
Gebruik: $f(x) = \frac{t}{n} \Rightarrow f'(x) = \frac{nat - tan}{n^2} = \frac{noemer \cdot afgeleide\ teller - teller \cdot afgeleide\ noemer}{noemer^2}$.

48a 💻 $f(x) = e^x + 2 \Rightarrow f'(x) = e^x$.

48b 💻 $f(x) = 2e^x + \frac{1}{x} = 2e^x + x^{-1} \Rightarrow f'(x) = 2e^x - 1x^{-2} = 2e^x - \frac{1}{x^2}$.

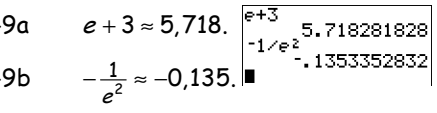
48c 💻 $f(x) = x \cdot e^x + 4 \Rightarrow f'(x) = 1 \cdot e^x + x \cdot e^x = e^x \cdot (1+x)$.

48d 💻 $f(x) = \frac{x}{e^x} \Rightarrow f'(x) = \frac{1 \cdot e^x - x \cdot e^x}{(e^x)^2} = \frac{e^x \cdot (1-x)}{e^x \cdot e^x} = \frac{1-x}{e^x}$.

48e 💻 $f(x) = \frac{2e^x}{x-1} \Rightarrow f'(x) = \frac{(x-1)\cdot 2e^x - 2e^x \cdot 1}{(x-1)^2} = \frac{2xe^x - 2e^x - 2e^x}{(x-1)^2} = \frac{2xe^x - 4e^x}{(x-1)^2} = \frac{2e^x \cdot (x-2)}{(x-1)^2}$.

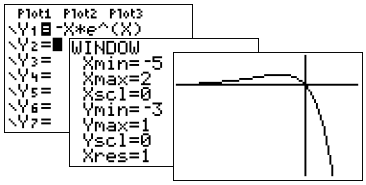
48f 💻 $f(x) = (2x-4) \cdot e^x \Rightarrow f'(x) = 2 \cdot e^x + (2x-4) \cdot e^x = e^x \cdot (2 + 2x - 4) = e^x \cdot (2x-2) = 2e^x \cdot (x-1)$.

49a $e + 3 \approx 5{,}718$.
49b $-\frac{1}{e^2} \approx -0{,}135$.
49c $e^3 \approx 20{,}086$.
49d $\frac{3e}{(e+2)^2} \approx 0{,}366$.
49e $1\frac{1}{3}e^2 \approx 9{,}852$.
49f $\frac{e^2}{e-3} \approx -26{,}229$.



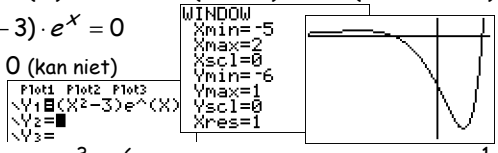
50a $f(x) = -x \cdot e^x \Rightarrow f'(x) = -1 \cdot e^x - x \cdot e^x = (-1-x) \cdot e^x$.
$f'(x) = 0 \Rightarrow (-1-x) \cdot e^x = 0 \Rightarrow -1 - x = 0 \vee e^x = 0$ (kan niet) $\Rightarrow x = -1$.
Maximum (zie plot) is $f(-1) = --1 \cdot e^{-1} = \frac{1}{e}$.

50b k: $y = ax$ met $a = f'(0) = (-1-0) \cdot e^0 = -1 \Rightarrow k$: $y = -x$.



51a $f(x) = 0$
$(x^2 - 3) \cdot e^x = 0$
$x^2 = 3 \vee e^x = 0$ (kan niet)
$x = -\sqrt{3} \vee x = \sqrt{3}$.

51b $f(x) = (x^2 - 3) \cdot e^x \Rightarrow f'(x) = 2x \cdot e^x + (x^2 - 3) \cdot e^x = (x^2 + 2x - 3) \cdot e^x$.
$f'(x) = 0 \Rightarrow (x^2 + 2x - 3) \cdot e^x = 0$
$x^2 + 2x - 3 = 0 \vee e^x = 0$ (kan niet)
$(x+3) \cdot (x-1) = 0$
$x = -3 \vee x = 1$.
Max. (zie plot) is $f(-3) = 6 \cdot e^{-3} = \frac{6}{e^3}$ en min. (zie plot) is $f(1) = -2 \cdot e^1 = -2e$.



51c Voor grote negatieve waarden van x nadert $f(x) = (x^2 - 3) \cdot e^x$ naar nul $\Rightarrow y = 0$ (de x-as) is horizontale asymptoot.

51d $f(x) = p$ heeft precies twee oplossingen voor $p = \frac{6}{e^3} \vee -2e < p \le 0$. (gebruik 51abc en de grafiek)

52 $f(x) = \frac{2e^x}{e^x + 1} \Rightarrow f'(x) = \frac{(e^x+1)\cdot 2e^x - 2e^x \cdot e^x}{(e^x+1)^2} = \frac{2e^{2x} + 2e^x - 2e^{2x}}{(e^x+1)^2} = \frac{2e^x}{(e^x+1)^2}$.
Stel k: $y = ax + b$ met $a = f'(1) = \frac{2e}{(e+1)^2}$.
k: $y = \frac{2e}{(e+1)^2}x + b$ door $P(1, \frac{2e}{e+1})$ $\Rightarrow \frac{2e}{e+1} = \frac{2e}{(e+1)^2} \cdot 1 + b \Rightarrow b = \frac{2e}{e+1} \cdot \frac{e+1}{e+1} - \frac{2e}{(e+1)^2} = \frac{2e^2 + 2e - 2e}{(e+1)^2} = \frac{2e^2}{(e+1)^2}$.
Dus k: $y = \frac{2e}{(e+1)^2}x + \frac{2e^2}{(e+1)^2}$.

k snijden met $y = 2$ geeft:

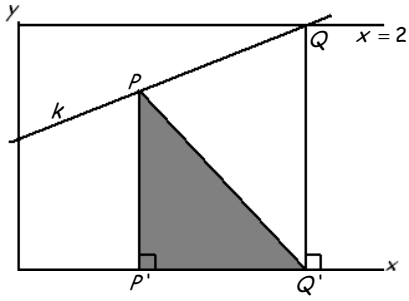
$\frac{2e}{(e+1)^2}x + \frac{2e^2}{(e+1)^2} = 2$

$\frac{2e}{(e+1)^2}x = 2 - \frac{2e^2}{(e+1)^2}$

$x = \frac{(e+1)^2}{2e} \cdot \left(2 - \frac{2e^2}{(e+1)^2}\right)$

$x = \frac{(e+1)^2}{2e} \cdot 2 - \frac{(e+1)^2}{2e} \cdot \frac{2e^2}{(e+1)^2}$

$x = \frac{(e+1)^2}{e} - \frac{2e^2}{2e} = \frac{e^2+2e+1}{e} - \frac{e^2}{e} = \frac{2e+1}{e} \Rightarrow Q(\frac{2e+1}{e}, 2)$.



$P'(1, 0)$ en $Q'(2 + \frac{1}{e}, 0)$.

$Opp.(\Delta PP'Q') = \frac{1}{2} \times basis \times hoogte$

$= \frac{1}{2} \cdot PQ' \cdot PP'$

$= \frac{1}{2} \cdot PQ' \cdot y_P$

$= \frac{1}{2} \cdot \left(2 + \frac{1}{e} - 1\right) \cdot \frac{2e}{e+1}$

$= \frac{1}{2} \cdot \left(1 + \frac{1}{e}\right) \cdot \frac{2e}{e+1}$

$= \frac{1}{2} \cdot \left(\frac{e}{e} + \frac{1}{e}\right) \cdot \frac{2e}{e+1}$

$= \frac{1}{2} \cdot \frac{e+1}{e} \cdot \frac{2e}{e+1} = 1$.

53 $f(x) = e^{ax+b} \Rightarrow f'(x) = e^{\boxed{ax+b}} \cdot a = a \cdot e^{ax+b}$.

54a $f(x) = e^{x^2+x} \Rightarrow f'(x) = e^{\boxed{x^2+x}} \cdot (2x+1) = (2x+1) \cdot e^{x^2+x}$.

54b $g(x) = x^2 + 2e^{3x} \Rightarrow g'(x) = 2x + 2e^{\boxed{3x}} \cdot 3 = 2x + 6e^{3x}$.

54c $h(x) = x \cdot e^{x^2} \Rightarrow h'(x) = 1 \cdot e^{x^2} + x \cdot e^{\boxed{x^2}} \cdot 2x = e^{x^2} + 2x^2 \cdot e^{x^2} = (1 + 2x^2) \cdot e^{x^2}$.

54d $j(x) = 3x \cdot e^{2x-1} \Rightarrow j'(x) = 3 \cdot e^{2x-1} + 3x \cdot e^{\boxed{2x-1}} \cdot 2 = 3 \cdot e^{2x-1} + 6x \cdot e^{2x-1} = (3 + 6x) \cdot e^{2x-1}$.

54e $k(x) = \frac{2e^{-x-1}}{x^2} \Rightarrow k'(x) = \frac{x^2 \cdot 2e^{\boxed{-x-1}} \cdot -1 - 2e^{-x-1} \cdot 2x}{(x^2)^2} = \frac{-2x^2e^{-x-1} - 4xe^{-x-1}}{x^4} = \frac{-2xe^{-x-1} \cdot (x+2)}{x^4} = \frac{-2e^{-x-1} \cdot (x+2)}{x^3}$.

54f $l(x) = \frac{e^{2x}}{e^{2x}+1} \Rightarrow l'(x) = \frac{(e^{2x}+1) \cdot e^{\boxed{2x}} \cdot 2 - e^{2x} \cdot e^{\boxed{2x}} \cdot 2}{(e^{2x}+1)^2} = \frac{2e^{4x} + 2e^{2x} - 2e^{4x}}{(e^{2x}+1)^2} = \frac{2e^{2x}}{(e^{2x}+1)^2}$.

55a $f(x) = \frac{1}{2}e^{2x} \Rightarrow f'(x) = \frac{1}{2}e^{\boxed{2x}} \cdot 2 = e^{2x}$.

Stel k: $y = ax + b$ met $a = f'(-1) = e^{-2}$.

k: $y = e^{-2}x + b$ door $A(-1, \frac{1}{2}e^{-2})$ $\Rightarrow \frac{1}{2}e^{-2} = e^{-2} \cdot -1 + b \Rightarrow b = 1\frac{1}{2}e^{-2}$.

Dus k: $y = e^{-2}x + 1\frac{1}{2}e^{-2}$.

$g(x) = \frac{1}{e^{x+3}} = e^{-x-3} \Rightarrow g'(x) = e^{\boxed{-x-3}} \cdot -1 = -e^{-x-3}$.

Stel l: $y = ax + b$ met $a = g'(-1) = -e^{-1-3} = -e^{-2}$.

l: $y = -e^{-2}x + b$ door $B(-1, e^{-2})$ $\Rightarrow e^{-2} = -e^{-2} \cdot -1 + b \Rightarrow b = 0$.

Dus l: $y = -e^{-2}x$.

k en l snijden geeft: $e^{-2}x + 1\frac{1}{2}e^{-2} = -e^{-2}x$

$2e^{-2}x = -1\frac{1}{2}e^{-2}$

$2x = -1\frac{1}{2}$

$x = -\frac{3}{4}$.

55b $h(x) = f(x) + g(x) \Rightarrow$

$h'(x) = f'(x) + g'(x) = e^{2x} - e^{-x-3}$ (zie 55a).

$h'(x) = 0 \Rightarrow e^{2x} - e^{-x-3} = 0$

$e^{2x} = e^{-x-3}$

$2x = -x - 3$

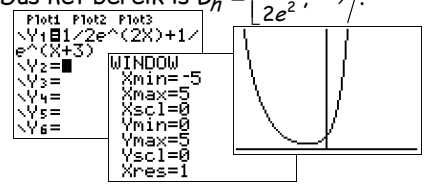
$3x = -3$

$x = -1$.

Min. (zie plot) is $h(-1) = \frac{1}{2}e^{-2} + \frac{1}{e^2}$

$= \frac{1}{2e^2} + \frac{2}{2e^2} = \frac{3}{2e^2}$.

Dus het bereik is $B_h = \left[\frac{3}{2e^2}, \rightarrow\right\rangle$.

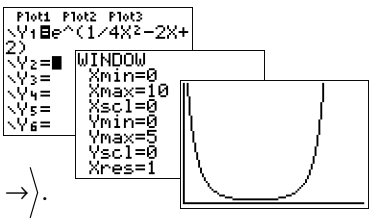


56a $f(x) = e^{\frac{1}{4}x^2-2x+2} \Rightarrow f'(x) = e^{\boxed{\frac{1}{4}x^2-2x+2}} \cdot (\frac{1}{2}x - 2) = (\frac{1}{2}x - 2) \cdot e^{\frac{1}{4}x^2-2x+2}$.

$f'(x) = 0 \Rightarrow (\frac{1}{2}x - 2) \cdot e^{\frac{1}{4}x^2-2x+2} = 0 \Rightarrow$

$\frac{1}{2}x - 2 = 0 \vee e^{\frac{1}{4}x^2-2x+2} = 0$ (kan niet) $\Rightarrow \frac{1}{2}x = 2 \Rightarrow x = 4$.

Minimum (zie plot) is $f(4) = e^{4-8+2} = e^{-2} = \frac{1}{e^2}$. Dus het bereik is $B_f = \left[\frac{1}{e^2}, \rightarrow\right\rangle$.



56b $Q(p, f(p)) = Q(p, e^{\frac{1}{4}p^2-2p+2})$ en $R(0, e^{\frac{1}{4}p^2-2p+2}) \Rightarrow O(\text{rechthoek } PQRS) = O(p) = OP \cdot OR = p \cdot e^{\frac{1}{4}p^2-2p+2}$.

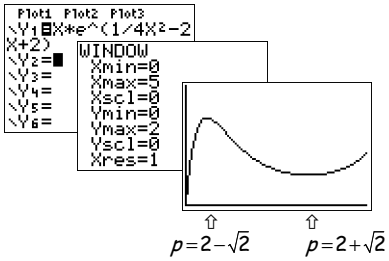
$O'(p) = 1 \cdot e^{\frac{1}{4}p^2-2p+2} + p \cdot e^{\frac{1}{4}p^2-2p+2} \cdot (\frac{1}{2}p - 2) = (\frac{1}{2}p^2 - 2p + 1) \cdot e^{\frac{1}{4}p^2-2p+2}$

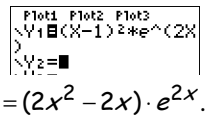
$O'(p) = 0 \Rightarrow (\frac{1}{2}p^2 - 2p + 1) \cdot e^{\frac{1}{4}p^2-2p+2} = 0$

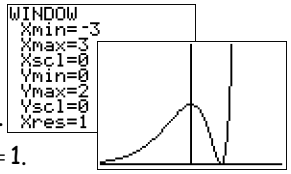
$\frac{1}{2}p^2 - 2p + 1 = 0$ (*abc*-formule) $\vee e^{\frac{1}{4}p^2-2p+2} = 0$ (kan niet)

$D = (-2)^2 - 4 \cdot \frac{1}{2} \cdot 1 = 4 - 2 = 2 \Rightarrow p = \frac{2 \pm \sqrt{2}}{1} = 2 \pm \sqrt{2}$.

De oppervlakte is maximaal (zie de plot hiernaast) voor $p = 2 - \sqrt{2}$.







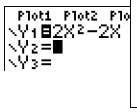
57a $f_1(x)=(x-1)^2\cdot e^{2x}=(x^2-2x+1)\cdot e^{2x}\Rightarrow$
$f_1'(x)=(2x-2)\cdot e^{2x}+(x^2-2x+1)\cdot e^{\boxed{2x}}\cdot 2=(2x-2+2x^2-4x+2)\cdot e^{2x}=(2x^2-2x)\cdot e^{2x}$.
$f_1'(x)=0\Rightarrow(2x^2-2x)\cdot e^{2x}\Rightarrow 2x^2-2x=0\vee e^{2x}=0$ (kan niet) $\Rightarrow 2x(x-1)=0\Rightarrow x=0\vee x=1$.
Maximum (zie de plot) is $f_1(0)=(-1)^2\cdot e^0=1\cdot 1=1$ en minimum (zie de plot) is $f_1(1)=(0)^2\cdot e^2=0$.

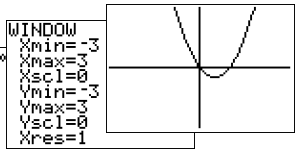
57b $f_a(x)=(x-a)^2\cdot e^{2x}=(x^2-2ax+a^2)\cdot e^{2x}\Rightarrow$
$f_a'(x)=(2x-2a)\cdot e^{2x}+(x^2-2ax+a^2)\cdot e^{\boxed{2x}}\cdot 2=(2x^2+(2-4a)x+2a^2-2a)\cdot e^{2x}$.
$f_a'(x)=0\Rightarrow 2x^2+(2-4a)x+2a^2-2a=0\vee e^{2x}=0$ (kan niet)
$x^2+(1-2a)x+a^2-a=0$ (*abc*-formule)
$D=(1-2a)^2-4\cdot 1\cdot(a^2-a)=1-4a+4a^2-4a^2+4a=1\Rightarrow x=\frac{-(1-2a)\pm\sqrt{1}}{2\cdot 1}=\frac{-1+2a\pm 1}{2}$.
$x=\frac{-1+2a+1}{2}=\frac{2a}{2}=a\vee x=\frac{-1+2a-1}{2}=\frac{2a-2}{2}=a-1$. Dus $x_A=a-1$ en $x_B=a$.

57c $x_B=a\Rightarrow y_B=f(x_B)=f(a)=(a-a)^2\cdot e^{2a}=0\cdot e^{2a}=0$. Dus de toppen van B liggen op de lijn $y=0$ (de x-as).

57d $x_A=a-1\Rightarrow y_A=f(x_A)=(a-1-a)^2\cdot e^{2(a-1)}=1\cdot e^{2(a-1)}=e^{2x_A}\Rightarrow$ de toppen van A liggen op de kromme $y=e^{2x}$.

57e C op de y-as $\Rightarrow x_C=0$. $\text{rc}_{\text{raaklijn in }C}=f_a'(0)=(2a^2-2a)\cdot e^0=(2a^2-2a)\cdot 1=2a^2-2a$.
$\text{rc}_{\text{raaklijn in }C}=0\Rightarrow 2a^2-2a=0\Rightarrow 2a(a-1)=0\Rightarrow a=0\vee a=1$.
$\text{rc}_{\text{raaklijn in }C}$ (zie plot) $<0\Rightarrow 0<a<1$.





58a $2^x=\left(e^{\,^e\log(2)}\right)^x=e^{\,^e\log(2)\cdot x}$ of $2^x=e^{\,^e\log(2^x)}=e^{x\cdot\,^e\log(2)}=e^{\,^e\log(2)\cdot x}$.

58b $2^x=e^{\,^e\log(2)\cdot x}\Rightarrow[2^x]'=e^{\boxed{\,^e\log(2)\cdot x}}\cdot\,^e\log(2)=\,^e\log(2)\cdot\left(e^{\,^e\log(2)}\right)^x=\,^e\log(2)\cdot 2^x$.

$e^{\cdots}$ en $\ln\ldots=\,^e\log\ldots$ heffen elkaar op
(zie ook de toetsen op de GR)

59a 💻 $\ln(e)=1$.

59b 💻 $\ln(e\cdot\sqrt{e})=\ln(e^1\cdot e^{\frac{1}{2}})=\ln(e^{1\frac{1}{2}})=1\frac{1}{2}$.

59c 💻 $\ln(\frac{1}{e})=\ln(e^{-1})=-1$.

59d 💻 $\ln(1)=\ln(e^0)=0$.

59e 💻 $3\cdot\ln(e\cdot\sqrt[3]{e})=3\cdot\ln(e^1\cdot e^{\frac{1}{3}})=3\cdot\ln(e^{1\frac{1}{3}})=\ln((e^{1\frac{1}{3}})^3)=\ln(e^4)=4$.

59f 💻 $\ln^2(e^3)=(\ln(e^3))^2=3^2=9$.

59g 💻 $\ln^3(e^2)=(\ln(e^2))^3=2^3=8$.

59h 💻 $e^{\ln(7)}+e^{2\cdot\ln(7)}=7+e^{\ln(7^2)}=7+7^2=7+49=56$.

59i 💻 $e^{\frac{1}{2}\cdot\ln(5)}=e^{\ln(5^{\frac{1}{2}})}=e^{\ln(\sqrt{5})}=\sqrt{5}$.

59j 💻 $e^{\ln(10)}\cdot e^{\ln(3)}=10\cdot 3=30$.

60a 💻 $e^{3x}=12$ (ln… nemen)
$3x=\ln(12)$ (:3)
$x=\frac{1}{3}\ln(12)$.

60b 💻 $5e^{2x}=60$ (:5)
$e^{2x}=12$ (ln… nemen)
$2x=\ln(12)$
$x=\frac{1}{2}\ln(12)$.

60c 💻 $6+e^{0,5x}=10$ (−6)
$e^{0,5x}=4$ (ln… nemen)
$0,5x=\ln(4)$ (×2)
$x=2\ln(4)$.

60d 💻 $\frac{3}{e^{2x}}=10$ (×e^{2x})
$3=10e^{2x}$ (:10)
$e^{2x}=\frac{3}{10}$ (ln… nemen)
$2x=\ln(\frac{3}{10})\Rightarrow x=\frac{1}{2}\ln(\frac{3}{10})$.

61a 💻 $2\cdot\ln(3)+\ln(4)=\ln(3^2)+\ln(4)=\ln(9\cdot 4)=\ln(36)$.

61b 💻 $\ln(20)-3\cdot\ln(2)=\ln(20)-\ln(2^3)=\ln(\frac{20}{8})=\ln(2\frac{1}{2})$.

61c 💻 $4+\ln(3)=\ln(e^4)+\ln(3)=\ln(e^4\cdot 3)=\ln(3e^4)$.

61d 💻 $1+\ln(10)=\ln(e^1)+\ln(10)=\ln(e\cdot 10)=\ln(10e)$.

61e 💻 $\frac{1}{2}+2\ln(6)=\ln(e^{\frac{1}{2}})+\ln(6^2)=\ln(\sqrt{e}\cdot 36)=\ln(36\sqrt{e})$.

61f 💻 $e+\ln(2)=\ln(e^e)+\ln(2)=\ln(e^e\cdot 2)=\ln(2e^e)$.

62a 💻 $\ln(x)=-1$ BV: $x>0$ ($e^{\cdots}$ nemen)
$x=e^{-1}=\frac{1}{e}$ (voldoet).

62b 💻 $4\ln(x)=2$ BV: $x>0$ (:4)
$\ln(x)=\frac{1}{2}$ ($e^{\cdots}$ nemen)
$x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet).

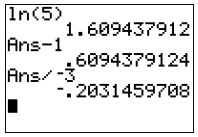
62c 💻 $\ln(3x)=3$ BV: $x>0$ ($e^{\cdots}$ nemen)
$3x=e^3$ (:3)
$x=\frac{1}{3}e^3$ (voldoet).

62d 💻 $\ln(-x+2)=-2$ BV: $-x+2>0\Rightarrow -x>-2\Rightarrow x<2$ ($e^{\cdots}$ nemen)
$-x+2=e^{-2}=\frac{1}{e^2}$ (−2)
$-x=-2+\frac{1}{e^2}$ (×−1) $\Rightarrow x=2-\frac{1}{e^2}$ (voldoet).

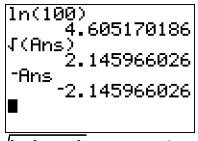
62e 💻 $\ln^2(x)=\frac{1}{4}$ BV: $x>0$
$\ln(x)=\pm\frac{1}{2}$ ($e^{\cdots}$ nemen)
$x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet) $\vee\ x=e^{-\frac{1}{2}}=\frac{1}{\sqrt{e}}$ (voldoet).

62f 💻 $\ln(x)=1+\ln(5)$ BV: $x>0$
$\ln(x)=\ln(e)+\ln(5)$
$\ln(x)=\ln(5e)$ ($e^{\cdots}$ nemen)
$x=5e$ (voldoet).

63a 💻 $4e^{1-3x} = 20$ $(:4)$
$e^{1-3x} = 5$ (ln... nemen)
$1-3x = \ln(5)$ (-1)
$-3x = -1+\ln(5)$ $(:-3)$
$x = \frac{-1+\ln(5)}{-3} \approx -0{,}203.$



63b 💻 $e^{x^2} = 100$ (ln... nemen)
$x^2 = \ln(100)$
$x = \pm\sqrt{\ln(100)}$
$x = \sqrt{\ln(100)} \approx 2{,}146 \vee x = -\sqrt{\ln(100)} \approx -2{,}146.$



64a $3x \cdot \ln(x) = 2 \cdot \ln(x)$ BV: $x > 0$
$3x \cdot \ln(x) - 2 \cdot \ln(x) = 0$
$(3x-2) \cdot \ln(x) = 0$
$3x = 2 \vee \ln(x) = 0$ ($e^{\cdots}$ nemen)
$x = \frac{2}{3}$ (voldoet) $\vee$ $x = e^0 = 1$ (voldoet).

64b $\ln^2(x) - \ln(x) = 0$ BV: $x > 0$ (stel $\ln(x) = t$)
$t^2 - t = 0$
$t \cdot (t-1) = 0$
$t = \ln(x) = 0 \vee t = \ln(x) = 1$ ($e^{\cdots}$ nemen)
$x = e^0 = 1$ (voldoet) $\vee$ $x = e^1 = e$ (voldoet).

64c $x^2 \cdot \ln(x+1) = 4 \cdot \ln(x+1)$ BV: $x > -1$
$x^2 \cdot \ln(x+1) - 4 \cdot \ln(x+1) = 0$
$(x^2-4) \cdot \ln(x+1) = 0$
$x^2 = 4 \vee \ln(x+1) = 0$ ($e^{\cdots}$ nemen)
$x = \pm 2 \vee x + 1 = e^0 = 1$
$x = 2$ (voldoet) $\vee$ $x = -2$ (vold. niet) $\vee$ $x = 0$ (voldoet).

64d $\ln^2(x) - 2\ln(x) - 3 = 0$ BV: $x > 0$ (stel $\ln(x) = t$)
$t^2 - 2t - 3 = 0$
$(t-3) \cdot (t+1) = 0$
$t = \ln(x) = 3 \vee t = \ln(x) = -1$ ($e^{\cdots}$ nemen)
$x = e^3$ (voldoet) $\vee$ $x = e^{-1} = \frac{1}{e}$ (voldoet).

64e $\ln(x+3) - \ln(x-1) = \ln(2)$ BV: $x > 1$
$\ln(x+3) = \ln(x-1) + \ln(2)$
$\ln(x+3) = \ln(2 \cdot (x-1))$ ($e^{\cdots}$ nemen)
$x + 3 = 2x - 2$
$-x = -5$
$x = 5$ (voldoet).

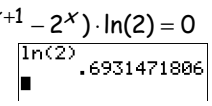
64f $2 \cdot \ln(x) = \ln(2) + \ln(x+4)$ BV: $x > 0$
$\ln(x^2) = \ln(2 \cdot (x+4))$ ($e^{\cdots}$ nemen)
$x^2 = 2x + 8$
$x^2 - 2x - 8 = 0$
$(x-4) \cdot (x+2) = 0$
$x = 4$ (voldoet) $\vee$ $x = -2$ (vold. niet).

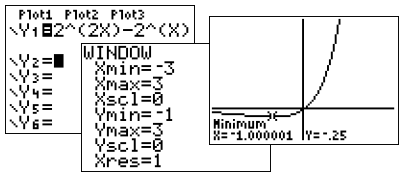
65a 💻 $f(x) = 3^{4x-2} \Rightarrow f'(x) = 3^{\boxed{4x-2}} \cdot \ln(3) \cdot 4 = 4 \cdot 3^{4x-2} \cdot \ln(3).$
65b 💻 $g(x) = (2x-1) \cdot 2^x \Rightarrow g'(x) = 2 \cdot 2^x + (2x-1) \cdot 2^x \cdot \ln(2) = (2 + (2x-1) \cdot \ln(2)) \cdot 2^x.$

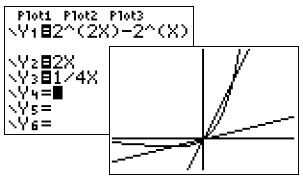
$f(x) = a^x \Rightarrow f'(x) = a^x \cdot \ln a.$

65c 💻 $h(x) = \frac{2^x+1}{2^x-1} \Rightarrow h'(x) = \frac{(2^x-1) \cdot 2^x \cdot \ln(2) - (2^x+1) \cdot 2^x \cdot \ln(2)}{(2^x-1)^2} = \frac{(2^x-1-2^x-1) \cdot 2^x \cdot \ln(2)}{(2^x-1)^2} = \frac{-2 \cdot 2^x \cdot \ln(2)}{(2^x-1)^2}.$

66a $f(x) = 2^{2x} - 2^x \Rightarrow f'(x) = 2^{\boxed{2x}} \cdot \ln(2) \cdot 2 - 2^x \cdot \ln(2) = (2^{2x} \cdot 2 - 2^x) \cdot \ln(2) = (2^{2x+1} - 2^x) \cdot \ln(2).$
$f'(x) = 0 \Rightarrow (2^{2x+1} - 2^x) \cdot \ln(2) = 0$
$2^{2x+1} - 2^x = 0$
$2^{2x+1} = 2^x$
$2x + 1 = x$
$x = -1.$







Minimum (zie plot) $f(-1) = 2^{-2} - 2^{-1} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \Rightarrow B_f = \left[-\frac{1}{4}, \rightarrow\right\rangle.$

66b $y = ax$ (lijn door de oorsprong en ook $f(0) = 1 - 1 = 0$) heeft twee oplossingen als $0 < a < f'(0) \vee a > f'(0)$.
$f'(0) = (2^1 - 2^0) \cdot \ln(2) = (2-1) \cdot \ln(2) = \ln(2)$ dus $0 < a < \ln(2) \vee a > \ln(2)$. ($y = ax$ met $a = \ln(2)$ raakt f in de oorsprong)

67a $f(x) = 2^{x-1} + 2^{-x-2} \Rightarrow f'(x) = 2^{\boxed{x-1}} \cdot \ln(2) \cdot 1 + 2^{\boxed{-x-2}} \cdot \ln(2) \cdot -1 = (2^{x-1} - 2^{-x-2}) \cdot \ln(2).$
$f'(x) = 0 \Rightarrow (2^{x-1} - 2^{-x-2}) \cdot \ln(2) = 0$
$2^{x-1} - 2^{-x-2} = 0$
$2^{x-1} = 2^{-x-2}$
$x - 1 = -x - 2$
$2x = -1$
$x = -\frac{1}{2}.$

Minimum (zie figuur 9.8) $f(-\frac{1}{2}) = 2^{-\frac{1}{2}-1} + 2^{\frac{1}{2}-2} = 2 \cdot 2^{-1\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}} \Rightarrow B_f = \left[\frac{1}{\sqrt{2}}, \rightarrow\right\rangle.$

67b $f'(x) = -\frac{1}{4}\cdot\ln(2) \Rightarrow (2^{x-1} - 2^{-x-2})\cdot\ln(2) = -\frac{1}{4}\cdot\ln(2)$

$2^{x-1} - 2^{-x-2} = -\frac{1}{4}$ $(\times 4)$

$2^2\cdot 2^{x-1} - 2^2\cdot 2^{-x-2} = -1$

$2^{x+1} - 2^{-x} = -1$

$2\cdot 2^x - \frac{1}{2^x} = -1$ (stel $2^x = t$)

$2t - \frac{1}{t} = -1$ $(\times t)$

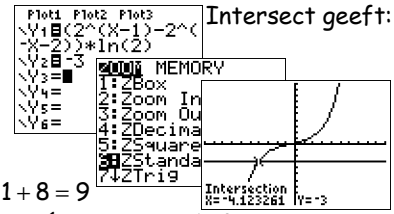
$2t^2 - 1 = -t$

$2t^2 + t - 1 = 0$ (*abc*-formule) ⇗

$D = 1^2 - 4\cdot 2\cdot -1 = 1 + 8 = 9$

$t = 2^x = \frac{-1+3}{4} = \frac{1}{2} = 2^{-1} \vee t = 2^x = \frac{-1-3}{4} = -1$ (kan niet)

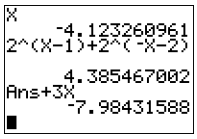
67c $f'(x) = -3 \Rightarrow (2^{x-1} - 2^{-x-2})\cdot\ln(2) = -3.$



Intersect geeft: $x_R \approx -4{,}1233$ en $y_R = f(x_R)$

Verder: $y_R = -3x_R + b \Rightarrow$

$y_R + 3x_R = b \approx -7{,}984.$



$x = -1 \Rightarrow y = f(-1) = 2^{-2} + 2^{-1} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$. Dus raakpunt $(-1, \frac{3}{4})$.

68a $e^{\ln(x)} = x$ (links en rechts de afgeleide nemen)

$e^{\boxed{\ln(x)}}\cdot[\ln(x)]' = 1$. Dus $e^{\ln(x)}\cdot[\ln(x)]' = 1$.

68b $\left.\begin{array}{l} e^{\ln(x)}\cdot[\ln(x)]' = 1 \\ e^{\ln(x)} = x \end{array}\right\} \Rightarrow x\cdot[\ln(x)]' = 1 \Rightarrow [\ln(x)]' = \frac{1}{x}.$

68c $g(x) = {}^2\log(x) = \frac{\ln(x)}{\ln(2)} = \frac{1}{\ln(2)}\cdot\ln(x) \Rightarrow g'(x) = \frac{1}{\ln(2)}\cdot\frac{1}{x} = \frac{1}{x\cdot\ln(2)}.$

69a $f(x) = \ln(6x) \Rightarrow f'(x) = \frac{1}{\boxed{6x}}\cdot 6 = \frac{1}{x}$

of $f(x) = \ln(6\cdot x) = \ln(6) + \ln(x) \Rightarrow f'(x) = 0 + \frac{1}{x} = \frac{1}{x}.$

69b $f(x) = \ln(2\cdot x) \Rightarrow f'(x) = \frac{1}{x}.$

$g(x) = \ln(x\cdot\sqrt{2}) \Rightarrow g'(x) = \frac{1}{x}.$

$h(x) = {}^2\log(3\cdot x) \Rightarrow h'(x) = \frac{1}{x\cdot\ln(2)}.$

70a $f(x) = \ln(x^6) \Rightarrow f'(x) = \frac{1}{\boxed{x^6}}\cdot 6x^5 = \frac{6x^5}{x^6} = \frac{6}{x}$

of $f(x) = \ln(x^6) = 6\cdot\ln(x) \Rightarrow f'(x) = 6\cdot\frac{1}{x} = \frac{6}{x}.$

70b $f(x) = \ln(x^2) \Rightarrow f'(x) = \frac{2}{x}.$

$g(x) = \ln(\frac{1}{x^3}) = \ln(x^{-3}) \Rightarrow g'(x) = \frac{-3}{x}.$

$h(x) = \ln(\frac{1}{x}) = \ln(x^{-1}) \Rightarrow h'(x) = \frac{-1}{x}.$

71a 💻 $f(x) = \frac{1-\ln(x)}{x} \Rightarrow f'(x) = \frac{x\cdot -\frac{1}{x} - (1-\ln(x))\cdot 1}{x^2} = \frac{-1-1+\ln(x)}{x^2} = \frac{\ln(x)-2}{x^2}.$

71b 💻 $f(x) = x\cdot\ln(x) \Rightarrow f'(x) = 1\cdot\ln(x) + x\cdot\frac{1}{x} = \ln(x) + 1.$

71c 💻 $f(x) = {}^2\log(4x-1) \Rightarrow f'(x) = \frac{1}{(\boxed{4x-1})\cdot\ln(2)}\cdot 4 = \frac{4}{(4x-1)\cdot\ln(2)}.$

71d 💻 $f(x) = \frac{\ln(3x)}{x} \Rightarrow f'(x) = \frac{x\cdot\frac{1}{x} - \ln(3x)\cdot 1}{x^2} = \frac{1-\ln(3x)}{x^2}.$

71e 💻 $f(x) = x\cdot\ln(x^3) \Rightarrow f'(x) = 1\cdot\ln(x^3) + x\cdot\frac{3}{x} = \ln(x^3) + 3.$

71f 💻 $f(x) = {}^3\log(x^2) \Rightarrow f'(x) = \frac{2}{x\cdot\ln(3)}.$

$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$

$f(x) = \ln(x^a) \Rightarrow f'(x) = \frac{a}{x}$

72a 💻 $f(x) = \ln(x^2 + x) \Rightarrow f'(x) = \frac{1}{\boxed{x^2+x}}\cdot(2x+1) = \frac{2x+1}{x^2+x}.$

72b 💻 $g(x) = \ln(2^x) \Rightarrow g'(x) = \frac{1}{\boxed{2^x}}\cdot 2^x\cdot\ln(2) = \ln(2)$ of $g(x) = \ln(2^x) = x\cdot\ln(2) = \ln(2)\cdot x \Rightarrow g'(x) = \ln(2)\cdot 1 = \ln(2).$

72c 💻 $h(x) = {}^2\log(x^2+1) \Rightarrow h'(x) = \frac{1}{(\boxed{x^2+1})\cdot\ln(2)}\cdot 2x = \frac{2x}{(x^2+1)\cdot\ln(2)}.$

72d 💻 $j(x) = \log(4x^2) \Rightarrow j'(x) = \frac{1}{\boxed{4x^2}\cdot\ln(10)}\cdot 8x = \frac{2}{x\cdot\ln(10)}$ of $j(x) = \log(4x^2) = \log(4) + \log(x^2) \Rightarrow j'(x) = \frac{2}{x\cdot\ln(10)}.$

73a $f(x) = x\cdot\ln^2(x) \Rightarrow f'(x) = 1\cdot\ln^2(x) + x\cdot 2\boxed{\ln(x)}\cdot\frac{1}{x} = \ln^2(x) + 2\cdot\ln(x).$

73b $g(x) = x^2\cdot{}^3\log(4x) \Rightarrow g'(x) = 2x\cdot{}^3\log(4x) + x^2\cdot\frac{1}{x\cdot\ln(3)} = 2x\cdot{}^3\log(4x) + \frac{x}{\ln(3)}.$

73c $h(x) = \log^2(4x) \Rightarrow h'(x) = 2\cdot\boxed{\log(4x)}\cdot\frac{1}{\boxed{4x}\cdot\ln(10)}\cdot 4 = \frac{2\cdot\log(4x)}{x\cdot\ln(10)}.$

73d $j(x) = \ln^2(4x^2+1) \Rightarrow j'(x) = 2\cdot\boxed{\ln(4x^2+1)}\cdot\frac{1}{\boxed{4x^2+1}}\cdot 8x = \frac{16x\cdot\ln(4x^2+1)}{4x^2+1}.$

74a $x^n = e^{\ln(x^n)} = e^{n\cdot\ln(x)}.$

74bc $f(x) = x^n = e^{n\cdot\ln(x)} \Rightarrow f'(x) = e^{\boxed{n\cdot\ln(x)}}\cdot n\cdot\frac{1}{x} = x^n\cdot n\cdot\frac{1}{x} = \frac{n\cdot x^n}{x} = nx^{n-1}.$

75a A op de x-as ($y=0$) $\Rightarrow f(x)=0$

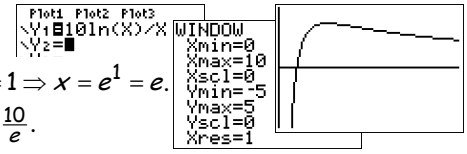
$\frac{10\ln(x)}{x}=0$ ($\Rightarrow$ teller $=0$) $\Rightarrow \ln(x)=0 \Rightarrow x=1 \Rightarrow A(1,0)$

$f(x)=\frac{10\ln(x)}{x}$ (BV: $x>0$) $\Rightarrow f'(x)=\frac{x\cdot 10\cdot\frac{1}{x}-10\ln(x)\cdot 1}{x^2}=\frac{10-10\ln(x)}{x^2}$

Stel de raaklijn: k: $y=ax+b$ met $a=f'(1)=\frac{10-10\ln(1)}{1^2}=\frac{10-0}{1}=10$

$y=10x+b$ door $A(1,0) \Rightarrow 0=10\cdot 1+b \Rightarrow b=-10$. Dus k: $y=10x-10$.

75b $f'(x)=0 \Rightarrow \frac{10-10\ln(x)}{x^2}=0$ ($\Rightarrow$ teller $=0$)



$10-10\ln(x)=0 \Rightarrow 10\ln(x)=10 \Rightarrow \ln(x)=1 \Rightarrow x=e^1=e$.

Maximum (zie plot) $f(e)=\frac{10\ln(e)}{e}=\frac{10\cdot 1}{e}=\frac{10}{e}$.

75c Stel $x_B=p$ dan is $x_C=2p$

$f(p)=f(2p)=q$ geeft

$\frac{10\ln(p)}{p}=\frac{10\ln(2p)}{2p}$

$10\ln(p)=5\ln(2p)$

$2\ln(p)=\ln(2p)$

$\ln(p^2)=\ln(2p)$

$p^2=2p$

$p=0$ (vold. niet) $\vee$ $p=2$ (voldoet)

$q=f(p)=f(2)=\frac{10\ln(2)}{2}=5\ln(2)$.

76a $x_A=\frac{1}{e} \Rightarrow y_A=f(\frac{1}{e})=\frac{\frac{1}{e}}{\ln(\frac{1}{e})}=\frac{\frac{1}{e}}{\ln(e^{-1})}=\frac{\frac{1}{e}}{-1}=-\frac{1}{e}$

$f(x)=\frac{x}{\ln(x)} \Rightarrow f'(x)=\frac{\ln(x)\cdot 1-x\cdot\frac{1}{x}}{\ln^2(x)}=\frac{\ln(x)-1}{\ln^2(x)}$

Stel k: $y=ax+b$ met $a=f'(\frac{1}{e})=\frac{\ln(\frac{1}{e})-1}{\ln^2(\frac{1}{e})}=\frac{-1-1}{(-1)^2}=-2$

$y=-2x+b$ door $A(\frac{1}{e},-\frac{1}{e}) \Rightarrow -\frac{1}{e}=-2\cdot\frac{1}{e}+b \Rightarrow b=-\frac{1}{e}+\frac{2}{e}=\frac{1}{e}$. Dus k: $y=-2x+\frac{1}{e}$.

76b $f'(x)=-6 \Rightarrow \frac{\ln(x)-1}{\ln^2(x)}=-6$ (BV: $x>0$)

$-6\cdot\ln^2(x)=\ln(x)-1$ (stel $\ln(x)=t$)

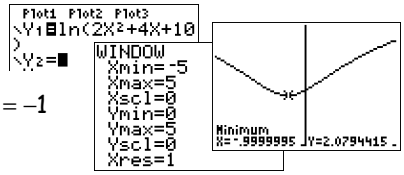
$-6t^2-t+1=0$ (*abc*-formule)

$D=(-1)^2-4\cdot -6\cdot 1=1+24=25 \Rightarrow t=\frac{1\pm\sqrt{25}}{2\cdot -6}$

$t=\ln(x)=\frac{1+5}{-12}=-\frac{1}{2} \vee t=\ln(x)=\frac{1-5}{-12}=\frac{1}{3}$

$\begin{cases} x=e^{-\frac{1}{2}}=\frac{1}{e^{\frac{1}{2}}}=\frac{1}{\sqrt{e}} \text{ (voldoet)} \\ y=f(e^{-\frac{1}{2}})=\frac{e^{-\frac{1}{2}}}{\ln(e^{-\frac{1}{2}})}=\frac{\frac{1}{\sqrt{e}}}{-\frac{1}{2}}=-\frac{2}{\sqrt{e}} \end{cases} \vee \begin{cases} x=e^{\frac{1}{3}}=\sqrt[3]{e} \text{ (voldoet)} \\ y=f(e^{\frac{1}{3}})=\frac{e^{\frac{1}{3}}}{\ln(e^{\frac{1}{3}})}=\frac{\sqrt[3]{e}}{\frac{1}{3}}=3\cdot\sqrt[3]{e}. \end{cases}$

77a $f(x)=\ln(2x^2+4x+10) \Rightarrow f'(x)=\frac{1}{2x^2+4x+10}\cdot(4x+4)=\frac{4x+4}{2x^2+4x+10}$



$f'(x)=0 \Rightarrow \frac{4x+4}{2x^2+4x+10}=0$ ($\Rightarrow$ teller $=0$) $\Rightarrow 4x+4=0 \Rightarrow 4x=-4 \Rightarrow x=-1$

minimum (zie plot) $f(-1)=\ln(2-4+10)=\ln(8) \Rightarrow B_f=[\ln(8),\ \rightarrow\rangle$.

77b $f'(x)=\frac{2}{5} \Rightarrow \frac{4x+4}{2x^2+4x+10}=\frac{2}{5}$

$4x^2+8x+20=20x+20$

$4x^2-12x=0$

$4x\cdot(x-3)=0$

$\begin{cases} x=0 \\ y=f(0)=\ln(10) \end{cases} \vee \begin{cases} x=3 \\ y=f(3)=\ln(2\cdot 3^2+4\cdot 3+10)=\ln(40). \end{cases}$

77c $f'(x)=1 \Rightarrow \frac{4x+4}{2x^2+4x+10}=1$

$2x^2+4x+10=4x+4$

$2x^2=-6$

$x^2=-3$ (kan niet).

78a $f(x)=g(x) \Rightarrow \ln(2x)=\ln(\frac{4}{x})$ BV: $x>0$

$2x=\frac{4}{x}$

$2x^2=4$

$x^2=2$

$\begin{cases} x=\sqrt{2} \text{ (voldoet)} \vee x=-\sqrt{2} \text{ (vold. niet)} \\ y=f(\sqrt{2})=\ln(2\sqrt{2}). \end{cases}$

78b A op de x-as ($y=0$) $\Rightarrow g(x)=0$

$\ln(\frac{4}{x})=0$ BV: $x>0 \Rightarrow \frac{4}{x}=e^0=1 \Rightarrow x=4$ (voldoet) $\Rightarrow A(4,0)$

$g(x)=\ln(\frac{4}{x})=\ln(4)-\ln(x) \Rightarrow g'(x)=0-\frac{1}{x}=-\frac{1}{x}$

Stel de raaklijn: $y=ax+b$ met $a=g'(4)=-\frac{1}{4}$

$y=-\frac{1}{4}x+b$ door $A(4,0) \Rightarrow 0=-\frac{1}{4}\cdot 4+b \Rightarrow b=1$.

Dus $y=-\frac{1}{4}x+1$.

78c $f(p)-g(p)=2 \vee g(p)-f(p)=2$

$\ln(2p)-\ln(\frac{4}{p})=2 \vee \ln(\frac{4}{p})-\ln(2p)=2$

$\ln(2p:\frac{4}{p})=2 \vee \ln(\frac{4}{p}:2p)=2$

$\ln(2p\cdot\frac{p}{4})=2 \vee \frac{2}{p^2}=e^2$

$\frac{p^2}{2}=e^2 \vee p^2=\frac{2}{e^2}$

$p^2=2e^2 \vee p=\frac{1}{e}\cdot\sqrt{2}$

$p=e\cdot\sqrt{2} \vee p=\frac{1}{e}\cdot\sqrt{2}$

($p=-e\sqrt{2}$ vold. niet) ($p=-\frac{1}{e}\sqrt{2}$ vold. niet)

78d $y_B=f(p)=\ln(2p)$ en $y_C=g(p)=\ln(\frac{4}{p})$

$y_M=\frac{y_B+y_C}{2}=\frac{\ln(2p)+\ln(\frac{4}{p})}{2}=\frac{\ln(2p\cdot\frac{4}{p})}{2}=\frac{\ln(8)}{2}$.

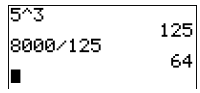
$\frac{\ln(8)}{2}$ is onafhankelijk van p,

dus het midden M van BC is onafhankelijk van p.

Diagnostische toets

D1a $^3\log(5)+2\cdot{}^3\log(2)={}^3\log(5)+{}^3\log(2^2)={}^3\log(5\cdot4)={}^3\log(20)$.

D1b $3-{}^2\log(5)={}^2\log(2^3)-{}^2\log(5)={}^2\log(8)-{}^2\log(5)={}^2\log(\frac{8}{5})$.



D1c $^2\log(8000)+3\cdot{}^2\log(\frac{1}{5})={}^2\log(8000)+{}^2\log((\frac{1}{5})^3)={}^2\log(8000)+{}^2\log(\frac{1}{125})={}^2\log(\frac{8000}{125})={}^2\log(64)={}^2\log(2^6)=6$.

D2a $2\cdot{}^2\log(x-1)=1+{}^2\log(18)$ BV: $x>1$
$^2\log((x-1)^2)={}^2\log(2^1)+{}^2\log(18)$
$^2\log((x-1)^2)={}^2\log(2\cdot18)$.
$(x-1)^2=36$
$x-1=\pm6$ (links en rechts $+1$)
$x=\pm6+1$
$x=6+1=7$ (voldoet) $\vee$ $x=-6+1=-5$ (vold. niet).

D2b $^2\log(x)=3-{}^2\log(x+2)$ BV: $x>0$
$^2\log(x)+{}^2\log(x+2)={}^2\log(2^3)$
$^2\log(x\cdot(x+2))={}^2\log(8)$.
$x\cdot(x+2)=8$.
$x^2+2x-8=0$
$(x+4)\cdot(x-2)=0$
$x=-4$ (vold. niet) $\vee$ $x=2$ (voldoet).

D3a $^2\log(x)-\frac{1}{2}\log(x-1)=3$ BV: $x>1$
$^2\log(x)+{}^2\log(x-1)={}^2\log(2^3)$
$^2\log(x\cdot(x-1))={}^2\log(8)$.
$x\cdot(x-1)=8$
$x^2-x-8=0$ (*abc*-formule)
$D=(-1)^2-4\cdot1\cdot-8=1+32=33\Rightarrow x=\frac{1\pm\sqrt{33}}{2\cdot1}$
$x=\frac{1+\sqrt{33}}{2}$ (voldoet) $\vee$ $x=3^x=\frac{1-\sqrt{33}}{2}<1$ (vold. niet).

D3b $\log^2(x)-5\cdot\log(x)=6$ BV: $x>0$
Stel $\log(x)$ tijdelijk t
$t^2-5t=6$
$t^2-5t-6=0$
$(t-6)\cdot(t+1)=0$
$t=\log(x)=6\vee t=\log(x)=-1$
$x=10^6=1\,000\,000$ (voldoet) $\vee$ $x=10^{-1}=\frac{1}{10}$ (voldoet).

D4a $3^x+6\cdot(\frac{1}{3})^x=5$
$3^x+6\cdot(\frac{1}{3^x})=5$ (links en rechts $\times3^x$)
$(3^x)^2+6\cdot1=5\cdot3^x$ (stel $3^x=t$)
$t^2+6=5t$
$t^2-5t+6=0$
$(t-2)\cdot(t-3)=0$
$t=3^x=2\vee t=3^x=3=3^1\Rightarrow x={}^3\log(2)\vee x=1$.

D4b $9^x=3^x+12$
$(3^2)^x=3^x+12$
$(3^x)^2-3^x-12=0$ (stel $3^x=t$)
$t^2-t-12=0$
$(t-4)\cdot(t+3)=0$
$t=3^x=4\vee t=3^x=-3$ (kan niet) $\Rightarrow x={}^3\log(4)$.

D4c $9^x=3^{x+1}+4$
$(3^2)^x=3^x\cdot3^1+4$
$(3^x)^2=3\cdot3^x+4$
$(3^x)^2-3\cdot3^x-4=0$ (stel $3^x=t$)
$t^2-3t-4=0$
$(t-4)\cdot(t+1)=0$
$t=3^x=4\vee t=3^x=-1$ (kan niet) $\Rightarrow x={}^3\log(4)$.

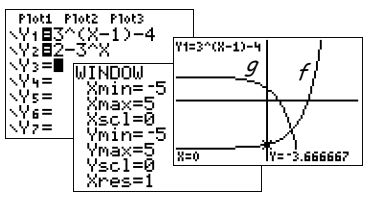
D4d $3^{x+2}+3^{2x+1}=12$
$3^x\cdot3^2+3^{2x}\cdot3^1=12$
$9\cdot3^x+3\cdot(3^x)^2-12=0$ (stel $3^x=t$ en deel door 3)
$t^2+3t-4=0$
$(t+4)\cdot(t-1)=0$
$t=3^x=-4$ (kan niet) $\vee$ $t=3^x=1=3^0\Rightarrow x=0$.

D5a $y=3^x\xrightarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{3}}f(x)=\frac{1}{3}\cdot3^x=3^{-1}\cdot3^x=3^{x-1}\xleftarrow{\text{translatie (1, 0)}}y=3^x$.

D5b $y={}^3\log(x)\xrightarrow{\text{translatie (0, }-2)}f(x)={}^3\log(x)-2={}^3\log(x)+{}^3\log(3^{-2})={}^3\log(x\cdot\frac{1}{9})={}^3\log(\frac{1}{9}\cdot x)$.
$y={}^3\log(x)\xrightarrow{\text{verm. t.o.v. de } y\text{-as met } 9}f(x)={}^3\log(\frac{1}{9}\cdot x)$.

D6a

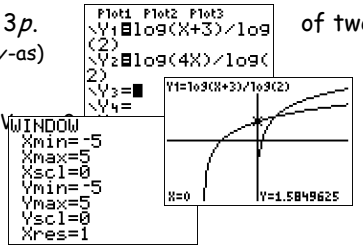
$f(p)-g(p)=2$	$\vee$	$g(p)-f(p)=2$
$3^{p-1}-4-(2-3^p)=2$	$\vee$	$2-3^p-(3^{p-1}-4)=2$
$3^{p-1}-4-2+3^p=2$	$\vee$	$2-3^p-3^{p-1}+4=2$
$3^{p-1}-6+3^p=2$ (links en rechts $\times3^1$)	$\vee$	$6-3^p-3^{p-1}=2$ (links en rechts $\times3^1$)
$3^p-18+3\cdot3^p=6$	$\vee$	$18-3\cdot3^p-3^p=6$
$4\cdot3^p=24$	$\vee$	$-4\cdot3^p=-12$
$3^p=6$	$\vee$	$3^p=3$
$p={}^3\log(6)$.	$\vee$	$p={}^3\log(3)=1$.



D6b $f(p)=g(p+1)=q$
$3^{p-1}-4=2-3^{p+1}$ (links en rechts $\times 3^1$)
$3^p-12=6-3\cdot 3^p\cdot 3^1$
$3^p-12=6-9\cdot 3^p$
$10\cdot 3^p=18$
$3^p=\frac{18}{10}=\frac{9}{5}$
$p={}^3\log(\frac{9}{5})\Rightarrow q=f(p)=3^p\cdot 3^{-1}-4=\frac{9}{5}\cdot\frac{1}{3}-4=-3\frac{2}{5}$.

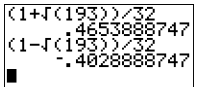
$\vee$ $g(p)=f(p+1)=q$
$\vee$ $2-3^p=3^{p+1-1}-4$
$\vee$ $2-3^p=3^p-4$
$\vee$ $-2\cdot 3^p=-6$
$\vee$ $3^p=3=3^1$
$\vee$ $p=1\Rightarrow q=g(p)=2-3^1=-1$.

D7a Stel $x_B=p>0$ dan $x_C=3p$.
(B en C beide rechts van de y-as)
$f(3p)=g(p)=q$
${}^2\log(3p+3)={}^2\log(4p)$ BV:
$3p+3=4p$
$-p=-3$
$p=3$ (voldoet)
$q=g(p)=g(3)={}^2\log(12)$.
Dus $q={}^2\log(12)$ of (zie hiernaast) $q={}^2\log(\frac{12}{5})$.



of tweede mogelijkheid: Stel $x_B=-p<0$ dan $x_C=p>0$.
(B links en C rechts van de y-as)
$f(-p)=g(p)=q$
${}^2\log(-p+3)={}^2\log(4p)$ BV: $0<p<3$
$-p+3=4p$
$-5p=-3$
$p=\frac{3}{5}$ (voldoet)
$q=g(p)=g(\frac{3}{5})={}^2\log(\frac{12}{5})$.

D7b E op f is het midden van DF.
$f(p)=2\cdot g(p)$
${}^2\log(p+3)=2\cdot {}^2\log(4p)$ BV: $p>0$
${}^2\log(p+3)={}^2\log((4p)^2)$
$p+3=16p^2$
$16p^2-p-3=0$ (abc-formule)



$D=(-1)^2-4\cdot 16\cdot -3=1+192=193\Rightarrow p=\frac{1\pm\sqrt{193}}{2\cdot 16}\Rightarrow p=\frac{1+\sqrt{193}}{32}\approx 0{,}47$ (voldoet) $\vee$ $p=\frac{1-\sqrt{193}}{32}$ (vold. niet).

D8a $\frac{3e^3-e^3}{e^2}=\frac{2e^3}{e^2}=2e$ of $\frac{3e^3-e^3}{e^2}=\frac{3e^3}{e^2}-\frac{e^3}{e^2}=3e-e=2e$.

D8b $(e^{3x}-5)^2=(e^{3x})^2-2\cdot 5\cdot e^{3x}+5^2=e^{6x}-10e^{3x}+25$.

D9a $3xe^x-e^x=0$
$(3x-1)\cdot e^x=0$
$3x=1\vee e^x=0$ (kan niet)
$x=\frac{1}{3}$.

D9b $e^{2x-1}-\sqrt[3]{e^2}=0$
$e^{2x-1}=\sqrt[3]{e^2}=e^{\frac{2}{3}}$
$2x-1=\frac{2}{3}$
$2x=1\frac{2}{3}=\frac{5}{3}$
$x=\frac{5}{6}$.

D9c $e^{4x}-e^{x+1}=0$
$e^{4x}=e^{x+1}$
$4x=x+1$
$3x=1$
$x=\frac{1}{3}$.

D9d $e^{2x}+2e^x=3$ (stel $e^x=t$)
$t^2+2t-3=0$
$(t+3)\cdot(t-1)=0$
$t=e^x=-3$ (kan niet) $\vee$ $t=e^x=1=e^0$
$x=0$.

D10a $f(x)=2e^x-3x^2\Rightarrow f'(x)=2e^x-6x$.

D10b $f(x)=\frac{x^2+1}{e^x}\Rightarrow f'(x)=\frac{e^x\cdot 2x-(x^2+1)\cdot e^x}{(e^x)^2}=\frac{(2x-x^2-1)\cdot e^x}{e^x\cdot e^x}=\frac{-x^2+2x-1}{e^x}$.

D10c $f(x)=(x^2+1)\cdot e^x\Rightarrow f'(x)=2x\cdot e^x+(x^2+1)\cdot e^x=(x^2+2x+1)\cdot e^x$.

D10d $f(x)=\frac{e^x}{x^2+1}\Rightarrow f'(x)=\frac{(x^2+1)\cdot e^x-e^x\cdot 2x}{(x^2+1)^2}=\frac{(x^2+2x+1)\cdot e^x}{(x^2+1)^2}$.

D10e $f(x)=x^2\cdot e^{2x-1}\Rightarrow f'(x)=2x\cdot e^{2x-1}+x^2\cdot e^{\boxed{2x-1}}\cdot 2=(2x^2+2x)\cdot e^{2x-1}$.

D10f $f(x)=e^{x^2+9}\Rightarrow f'(x)=e^{\boxed{x^2+9}}\cdot 2x=2x\cdot e^{x^2+9}$.

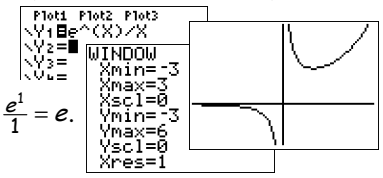
D11a $f(x)=\frac{e^x}{x}$ BV: $x\neq 0 \Rightarrow f'(x)=\frac{x\cdot e^x-e^x\cdot 1}{x^2}=\frac{(x-1)\cdot e^x}{x^2}$.

$f'(x)=0 \Rightarrow \frac{(x-1)\cdot e^x}{x^2}=0 \Rightarrow$ (teller = 0 en noemer ≠ 0)

$(x-1)\cdot e^x=0$

$x=1 \vee e^x=0$ (kan niet).

Minimum (zie plot) is $f(1)=\frac{e^1}{1}=e$.



D11b Stel l: $y=ax+b$ met $a=f'(2)=\frac{(2-1)\cdot e^2}{2^2}=\frac{e^2}{4}$.

l: $y=\frac{1}{4}e^2x+b$ door $A(2,\ \frac{1}{2}e^2)$ $\Rightarrow \frac{1}{2}e^2=\frac{1}{4}e^2\cdot 2+b \Rightarrow b=0$.

Dus l: $y=\frac{1}{4}e^2x$.

D12a $\ln(e^3\cdot\sqrt{e})=\ln(e^3\cdot e^{\frac{1}{2}})=\ln(e^{3\frac{1}{2}})=3\frac{1}{2}$.

D12b $\ln(\frac{1}{e^2})=\ln(e^{-2})=-2$.

D13a $4+\ln(3)=\ln(e^4)+\ln(3)=\ln(e^4\cdot 3)=\ln(3e^4)$.

D13b $\ln(10)-4\ln(2)=\ln(10)-\ln(2^4)=\ln(\frac{10}{16})=\ln(\frac{5}{8})$.

D14a $2\ln(5x)=16$ BV: $x>0$

$\ln(5x)=8$

$5x=e^8$

$x=\frac{1}{5}\cdot e^8$ (voldoet).

D14b $\ln^2(5x)=16$ BV: $x>0$

$\ln(5x)=\pm 4$

$5x=e^4 \vee 5x=-e^4$

$x=\frac{1}{5}\cdot e^4$ (voldoet) $\vee$ $x=\frac{1}{5}\cdot -e^4$ (vold. niet).

D14c $2\ln^2(x)-\ln(x)=0$ BV: $x>0$ (stel $\ln(x)=t$)

$2t^2-t=0$

$t\cdot(2t-1)=0$

$t=0 \vee 2t=1$

$t=\ln(x)=0 \vee t=\ln(x)=\frac{1}{2}$

$x=e^0=1$ (voldoet) $\vee$ $x=e^{\frac{1}{2}}=\sqrt{e}$ (voldoet).

D14d $\ln(9x+1)-\ln(x+2)=\ln(4)$ BV: $x>-\frac{1}{9}$

$\ln(9x+1)=\ln(4)+\ln(x+2)$

$9x+1=4\cdot(x+2)$

$9x+1=4x+8$

$5x=7$

$x=\frac{7}{5}=1\frac{2}{5}$ (voldoet).

D15a $f(x)=2^{3x-4} \Rightarrow f'(x)=2^{\boxed{3x-4}}\cdot\ln(2)\cdot 3=3\cdot\ln(2)\cdot 2^{3x-4}$.

D15b $f(x)=x\cdot 3^x \Rightarrow f'(x)=1\cdot 3^x+x\cdot 3^x\cdot\ln(3)=3^x\cdot(1+x\cdot\ln(3))$.

D15c $f(x)=\ln(x\cdot\sqrt[3]{x})=\ln(x\cdot x^{\frac{1}{3}})=\ln(x^{1\frac{1}{3}})=1\frac{1}{3}\cdot\ln(x) \Rightarrow f'(x)=1\frac{1}{3}\cdot\frac{1}{x}=\frac{4}{3x}$.

D15d $f(x)={}^2\log(4\cdot x)={}^2\log(4)+{}^2\log(x) \Rightarrow f'(x)=\frac{1}{x\cdot\ln(2)}$.

D15e $f(x)={}^3\log(5x-6) \Rightarrow f'(x)=\frac{1}{(\boxed{5x-6})\cdot\ln(3)}\cdot 5=\frac{5}{(5x-6)\cdot\ln(3)}$.

D15f $f(x)=\ln(3x^2+3) \Rightarrow f'(x)=\frac{1}{(\boxed{3x^2+3})}\cdot 6x=\frac{6x}{3\cdot(x^2+1)}=\frac{2x}{x^2+1}$.

D16a $f(x)=3^{x-1}+3^{-x+1} \Rightarrow$

$f'(x)=3^{\boxed{x-1}}\cdot\ln(3)\cdot 1+3^{\boxed{-x+1}}\cdot\ln(3)\cdot -1=(3^{x-1}-3^{-x+1})\cdot\ln(3)$.

$f'(x)=0 \Rightarrow (3^{x-1}-3^{-x+1})\cdot\ln(3)=0$

$3^{x-1}-3^{-x+1}=0$

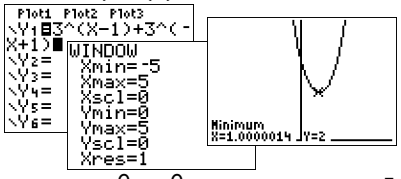
$3^{x-1}=3^{-x+1}$

$x-1=-x+1$

$2x=2$

$x=1$

minimum (zie plot) is $f(1)=3^0+3^0=1+1=2 \Rightarrow B_f=[2,\ \rightarrow\rangle$.



D16b $f'(x)=(3^{x-1}-3^{-x+1})\cdot\ln(3)=\frac{8}{3}\cdot\ln(3)$

$3^{x-1}-3^{-x+1}=\frac{8}{3}$ $(\times 3)$

$3^x-3^{-x+2}=8$ $(\times 3^x)$

$3^{2x}-3^{+2}=8\cdot 3^x$ (stel $3^x=t$)

$t^2-8t-9=0$

$(t-9)\cdot(t+1)=0$

$t=3^x=9=3^2 \vee t=3^x=-1$ (kan niet)

$x=2 \Rightarrow y=f(2)=3^{2-1}+3^{-2+1}=3+\frac{1}{3}=3\frac{1}{3}$.

D17a $f(x)=0$ BV: $x>0 \Rightarrow \frac{\ln(x)}{x}=0$ (teller = 0 $\Rightarrow$)

$\ln(x)=0 \Rightarrow x=e^0=1$ (voldoet)

$f(x)=\frac{\ln(x)}{x} \Rightarrow f'(x)=\frac{x\cdot\frac{1}{x}-\ln(x)\cdot 1}{x^2}=\frac{1-\ln(x)}{x^2}$.

k: $y=ax+b$ met $a=f'(1)=\frac{1-\ln(1)}{1^2}=\frac{1-0}{1}=1$.

$y=x+b$ door $(1,0) \Rightarrow 0=1+b \Rightarrow b=-1$

raaklijn: $y=x-1$.

D17b $f'(x)=0 \Rightarrow \frac{1-\ln(x)}{x^2}=0$ (teller = 0 $\Rightarrow$) BV: $x>0$

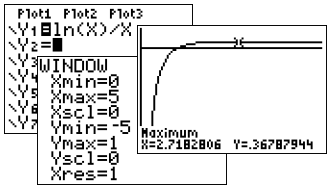
$1-\ln(x)=0$

$\ln(x)=1$

$x=e^1=e$ (voldoet)

maximum (zie plot) is $f(e)=\frac{\ln(e)}{e}=\frac{\ln(e^1)}{e}=\frac{1}{e}$.

Dus $B_f=\langle\leftarrow,\ \frac{1}{e}]$.



Gemengde opgaven 9. Exponentiële en logaritmische functies

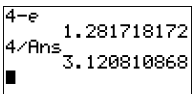
G1a $9^x = 3^x + 2$
$3^{2x} = 3^x + 2$ (stel $3^x = t$)
$t^2 - t - 2 = 0$
$(t-2)\cdot(t+1) = 0$
$t = 3^x = 2 \vee t = 3^x = -1$ (kan niet)
$x = {}^3\log(2)$.

G1b $\log^2(x) + 1 = 2\frac{1}{2}\log(x)$ BV: $x > 0$ (stel $\log(x) = t$)
$t^2 - 2\frac{1}{2}t + 1 = 0$
$(t-2)\cdot(t-\frac{1}{2}) = 0$
$t = \log(x) = 2 \vee t = \log(x) = \frac{1}{2}$
$x = 10^2 = 100$ (voldoet) $\vee$ $x = 10^{\frac{1}{2}} = \sqrt{10}$ (voldoet).

G1c $\frac{e^x}{e^x - 2} = 2$
$2e^x - 4 = e^x$
$e^x = 4$
$x = \ln(4)$.

G1d $\ln(3x+2) = \frac{1}{2}$ BV: $x > -\frac{2}{3}$
$\ln(3x+2) = \ln(e^{\frac{1}{2}})$
$3x + 2 = \sqrt{e}$
$3x = -2 + \sqrt{e}$
$x = -\frac{2}{3} + \frac{1}{3}\cdot\sqrt{e}$ (voldoet).

G1e $\ln(4x) - \ln(x+4) = 1$ BV: $x > 0$
$\ln(4x) = \ln(x+4) + \ln(e)$
$4x = e\cdot(x+4)$
$4x = ex + 4e$
$(4-e)\cdot x = 4e$
$x = \frac{4e}{4-e}$ (voldoet).



G1f $\ln^2(x-2) = 4$ BV: $x > 2$ (stel $\ln(x-2) = t$)
$t^2 = 4$
$t = \ln(x-2) = 2 \vee t = \ln(x-2) = -2$
$x - 2 = e^2 \vee x - 2 = e^{-2} = \frac{1}{e^2}$
$x = 2 + e^2$ (voldoet) $\vee$ $x = 2 + \frac{1}{e^2}$ (voldoet).

G1g $3\cdot 2^{2x+1} + 1 = 5\cdot 2^x$
$3\cdot 2^{2x}\cdot 2^1 + 1 = 5\cdot 2^x$ (stel $2^x = t$)
$6t^2 - 5t + 1 = 0$ (*abc*-formule)
$D = (-5)^2 - 4\cdot 6\cdot 1 = 25 - 24 = 1$
$t = \frac{5 \pm \sqrt{1}}{2\cdot 6}$
$t = 2^x = \frac{5+1}{12} = \frac{1}{2} = 2^{-1} \vee t = 2^x = \frac{5-1}{12} = \frac{1}{3}$
$x = -1 \vee x = {}^2\log(\frac{1}{3})$.

G1h $x\cdot 2^{-x+1} = 4x\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 4\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 2^2\cdot 2^{-3x+1}$
$x = 0 \vee 2^{-x+1} = 2^{-3x+3}$
$x = 0 \vee -x + 1 = -3x + 3$
$x = 0 \vee 2x = 2$
$x = 0 \vee x = 1$.

G2a $f(x) = x^2\cdot e^{x-1} \Rightarrow f'(x) = 2x\cdot e^{x-1} + x^2\cdot e^{\boxed{x-1}}\cdot 1 = (x^2+2x)\cdot e^{x-1}$.

G2b $g(x) = \ln^2(x) + \ln(x^2) \Rightarrow g'(x) = 2\cdot\boxed{\ln(x)}\cdot\frac{1}{x} + \frac{2}{x} = \frac{2\ln(x)+2}{x}$.

G2c $h(x) = {}^2\log(x^3 - x^2) \Rightarrow h'(x) = \frac{1}{(x^3-x^2)\cdot\ln(2)}\cdot(3x^2-2x) = \frac{3x^2-2x}{(x^3-x^2)\cdot\ln(2)}$.

G2d $j(x) = \ln(\ln(2x)) \Rightarrow j'(x) = \frac{1}{\boxed{\ln(2x)}}\cdot\frac{1}{x} = \frac{1}{x\cdot\ln(2x)}$.

$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(x^a) \Rightarrow f'(x) = \frac{a}{x}$
$f(x) = {}^g\log x \Rightarrow f'(x) = \frac{1}{x\cdot\ln(g)}$

G3a $f(x) = 2\cdot e^x \xrightarrow{\text{translatie } (3,0)} y = 2\cdot e^{x-3} = 2\cdot e^x\cdot e^{-3} = \frac{1}{e^3}\cdot 2\cdot e^x \xleftarrow{\text{verm. t.o.v. de } x\text{-as met } \frac{1}{e^3}} f(x) = 2\cdot e^x$.

G3b $g(x) = \ln(2x) \xrightarrow{\text{verm. t.o.v. de } y\text{-as met } 3} y = \ln(\frac{1}{3}\cdot 2x) = \ln(\frac{1}{3}) + \ln(2x) \xleftarrow{\text{translatie } (0, \ln(\frac{1}{3}))} g(x) = \ln(2x)$.

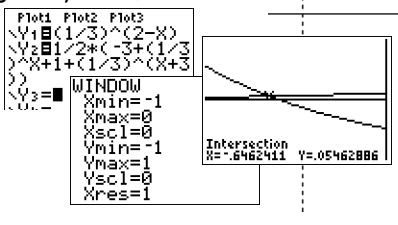
G4a Stel de snijpunten van de lijn $y = p$ met de grafieken van f, g en h zijn rescpectievelijk A, B en C.

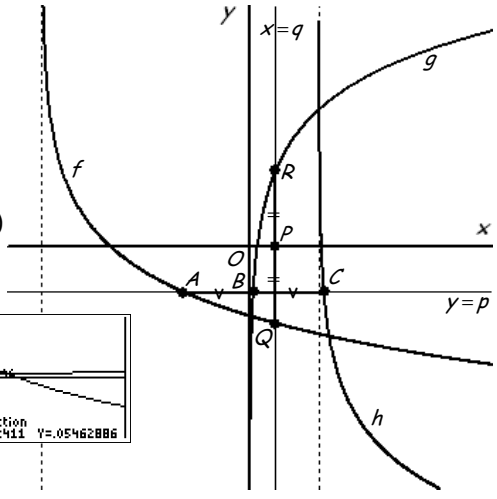
$f(x) = p \Rightarrow {}^{\frac{1}{3}}\log(x+3) = p$ BV: $x > -3$
$x + 3 = (\frac{1}{3})^p$
$x_A = -3 + (\frac{1}{3})^p$ (voldoet)

$h(x) = p \Rightarrow -3 + {}^{\frac{1}{3}}\log(x-1) = p$ BV: $x > 1$
${}^{\frac{1}{3}}\log(x-1) = p + 3$
$x - 1 = (\frac{1}{3})^{p+3}$
$x_C = 1 + (\frac{1}{3})^{p+3}$ (voldoet)

$g(x) = p \Rightarrow 2 - {}^{\frac{1}{3}}\log(x) = p$ BV: $x > 0$
${}^{\frac{1}{3}}\log(x) = 2 - p$
$x_B = (\frac{1}{3})^{2-p}$ (voldoet)

$AB = BC \Rightarrow x_B = \frac{1}{2}\cdot(x_A + x_C)$
$(\frac{1}{3})^{2-p} = \frac{1}{2}\cdot(-3 + (\frac{1}{3})^p + 1 + (\frac{1}{3})^{p+3})$
intersect geeft $p \approx -0{,}65$.





G4b P het midden van $QR \Rightarrow y_Q + y_R = 0$
$f(q) + g(x) = 0$
${}^{\frac{1}{3}}\log(q+3) + 2 - {}^{\frac{1}{3}}\log(q) = 0$ BV: $q > 0$
${}^{\frac{1}{3}}\log(q+3) + {}^{\frac{1}{3}}\log((\frac{1}{3})^2) = {}^{\frac{1}{3}}\log(q)$
${}^{\frac{1}{3}}\log(\frac{1}{9}\cdot(q+3)) = {}^{\frac{1}{3}}\log(q) \Rightarrow \frac{1}{9}\cdot(q+3) = q$ $(\times 9) \Rightarrow q + 3 = 9q \Rightarrow -8q = -3 \Rightarrow q = \frac{3}{8}$ (voldoet).

G5a $f_2(x)=\frac{2+2\ln(x)}{x}$ BV: $x>0$

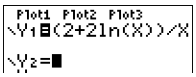
$f_2'(x)=\frac{x\cdot 2\cdot\frac{1}{x}-(2+2\ln(x))\cdot 1}{x^2}=\frac{2-2-2\ln(x)}{x^2}=\frac{-2\ln(x)}{x^2}$

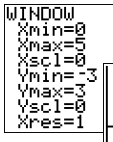
$f_2'(x)=0\Rightarrow\frac{-2\ln(x)}{x^2}=0$ (teller = 0)

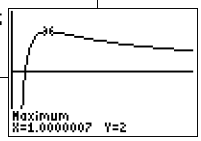
$-2\ln(x)=0$

$\ln(x)=0$

$x=e^0=1$ (voldoet)







$x_{top}=1$ en $y_{top}=f_2(1)=\frac{2+2\ln(1)}{1}=\frac{2+0}{1}=2$.

G5b $f_p(3)-f_{-p}(3)=4$ $\vee$ $f_{-p}(3)-f_p(3)=4$

$\frac{2+p\ln(3)}{3}-\frac{2-p\ln(3)}{3}=4$ $\vee$ $\frac{2-p\ln(3)}{3}-\frac{2+p\ln(3)}{3}=4$

$\frac{2+p\ln(3)-2+p\ln(3)}{3}=4$ $\vee$ $\frac{2-p\ln(3)-2-p\ln(3)}{3}=4$

$\frac{2p\ln(3)}{3}=4$ $\vee$ $\frac{-2p\ln(3)}{3}=4$

$2p\ln(3)=12$ $\vee$ $-2p\ln(3)=12$

$p\ln(3)=6$ $\vee$ $p\ln(3)=-6$

$p=\frac{6}{\ln(3)}$ $\vee$ $p=-\frac{6}{\ln(3)}$.

G6a $f(x)=g(x)\Rightarrow e^{\frac{1}{2}x-1}=e^{-x+1}\Rightarrow\frac{1}{2}x-1=-x+1\Rightarrow 1\frac{1}{2}x=2\Rightarrow x_A=2\cdot\frac{2}{3}=\frac{4}{3}$ en $y_A=f(\frac{4}{3})=e^{\frac{2}{3}-1}=e^{-\frac{1}{3}}=\frac{1}{e^{\frac{1}{3}}}=\frac{1}{\sqrt[3]{e}}$.

$f(x)=e^{\frac{1}{2}x-1}\Rightarrow f'(x)=e^{\boxed{\frac{1}{2}x-1}}\cdot\frac{1}{2}=\frac{1}{2}e^{\frac{1}{2}x-1}$

k: $y=ax+b$ met $a=f'(\frac{4}{3})=\frac{1}{2}e^{\frac{2}{3}-1}=\frac{1}{2}e^{-\frac{1}{3}}$

k: $y=\frac{1}{2}e^{-\frac{1}{3}}x+b$ door $A(\frac{4}{3},e^{-\frac{1}{3}})$

$e^{-\frac{1}{3}}=\frac{1}{2}e^{-\frac{1}{3}}\cdot\frac{4}{3}+b$

$e^{-\frac{1}{3}}=\frac{2}{3}e^{-\frac{1}{3}}+b$

$b=\frac{1}{3}e^{-\frac{1}{3}}\Rightarrow k$: $y=\frac{1}{2}e^{-\frac{1}{3}}x+\frac{1}{3}e^{-\frac{1}{3}}$

k snijden met de x-as ($y=0$):

$\frac{1}{2}e^{-\frac{1}{3}}x+\frac{1}{3}e^{-\frac{1}{3}}=0$

$\frac{1}{2}e^{-\frac{1}{3}}x=-\frac{1}{3}e^{-\frac{1}{3}}$

$x_B=-\frac{2}{3}$

$g(x)=e^{-x+1}\Rightarrow g'(x)=e^{\boxed{-x+1}}\cdot -1=-e^{-x+1}$

l: $y=ax+b$ met $a=g'(\frac{4}{3})=-e^{-\frac{4}{3}+1}=-e^{-\frac{1}{3}}$

l: $y=-e^{-\frac{1}{3}}x+b$ door $A(\frac{4}{3},e^{-\frac{1}{3}})$

$e^{-\frac{1}{3}}=-e^{-\frac{1}{3}}\cdot\frac{4}{3}+b$

$e^{-\frac{1}{3}}=-\frac{4}{3}e^{-\frac{1}{3}}+b$

$b=2\frac{1}{3}e^{-\frac{1}{3}}\Rightarrow l$: $y=-e^{-\frac{1}{3}}x+2\frac{1}{3}e^{-\frac{1}{3}}$

l snijden met de x-as ($y=0$):

$-e^{-\frac{1}{3}}x+2\frac{1}{3}e^{-\frac{1}{3}}=0$

$-e^{-\frac{1}{3}}x=-2\frac{1}{3}e^{-\frac{1}{3}}$

$x_C=2\frac{1}{3}$.

$O_{\Delta ABC}=\frac{1}{2}\cdot BC\cdot y_A=\frac{1}{2}\cdot(x_C-x_B)\cdot y_A=\frac{1}{2}\cdot(2\frac{1}{3}--\frac{2}{3})\cdot y_A=\frac{1}{2}\cdot 3\cdot\frac{1}{\sqrt[3]{e}}=\frac{3}{2}\cdot\frac{1}{\sqrt[3]{e}}=\frac{3}{2\cdot\sqrt[3]{e}}$.

G6b Stel $x=r$ de x-coördinaat van het linker snijpunt met $y=p$. Nu is $PQ=3$ als:

$f(r)=g(r+3)=p$ $\vee$ $g(r)=f(r+3)=p$

$e^{\frac{1}{2}r-1}=e^{-(r+3)+1}$ $\vee$ $e^{-r+1}=e^{\frac{1}{2}(r+3)-1}$

$\frac{1}{2}r-1=-r-3+1$ $\vee$ $-r+1=\frac{1}{2}q+1\frac{1}{2}-1$

$\frac{1}{2}r-1=-r-2$ $\vee$ $-r+1=\frac{1}{2}q+\frac{1}{2}$

$1\frac{1}{2}r=-1$ $\vee$ $1\frac{1}{2}r=-\frac{1}{2}$

$r=-1\cdot\frac{2}{3}=\frac{2}{3}$ $\vee$ $r=-\frac{1}{2}\cdot\frac{2}{3}=-\frac{1}{3}$

$p=f(r)=f(\frac{2}{3})=e^{\frac{1}{3}-1}=e^{-\frac{2}{3}}=\frac{1}{e^{\frac{2}{3}}}=\frac{1}{\sqrt[3]{e^2}}$ $\vee$ $p=g(r)=g(-\frac{1}{3})=e^{1\frac{1}{3}}=e^1\cdot e^{\frac{1}{3}}=e\cdot\sqrt[3]{e}$.

$PQ<3$ voor $\frac{1}{\sqrt[3]{e^2}}<p<e\cdot\sqrt[3]{e}$. (gebruik de uitkomsten hieboven en figuur G.1 in het boek)

G6c Stel $x_F=s$ dan is $x_E=4s$

$g(s)=f(4s)=q\Rightarrow e^{-s+1}=e^{2s-1}\Rightarrow -s+1=2s-1\Rightarrow -3s=-2\Rightarrow s=\frac{2}{3}\Rightarrow q=g(s)=g(\frac{2}{3})=e^{-\frac{2}{3}+1}=e^{\frac{1}{3}}=\sqrt[3]{e}$.

G7a $f(x)=g(x)\Rightarrow\ln(4x)=\ln(\frac{1}{x})$ BV: $x>0\Rightarrow 4x=\frac{1}{x}\Rightarrow 4x^2=1\Rightarrow x^2=\frac{1}{4}\Rightarrow x_A=+\frac{1}{2}$ en $y_A=f(\frac{1}{2})=\ln(2)$.

$f(x)=\ln(4x)\Rightarrow f'(x)=\frac{1}{x}$

k: $y=ax+b$ met $a=f'(\frac{1}{2})=\frac{1}{\frac{1}{2}}=1\cdot\frac{2}{1}=2$

k: $y=2x+b$ door $A(\frac{1}{2},\ln(2))$

$\ln(2)=2\cdot\frac{1}{2}+b$

$b=\ln(2)-1$

k: $y=2x+\ln(2)-1$.

$y=2x+\ln(2)-1$ snijdt de y-as ($x=0$) in $(0,\ln(2)-1)$ en

$g(x)=\ln(\frac{1}{x})=\ln(x^{-1})=-\ln x\Rightarrow g'(x)=-\frac{1}{x}$.

l: $y=ax+b$ met $a=g'(\frac{1}{2})=-\frac{1}{\frac{1}{2}}=-1\cdot\frac{2}{1}=-2$

l: $y=-2x+b$ door $A(\frac{1}{2},\ln(2))$

$\ln(2)=-2\cdot\frac{1}{2}+b$

$b=\ln(2)+1$

l: $y=-2x+\ln(2)+1$.

$y=-2x+\ln(2)+1$ snijdt de y-as ($x=0$) in $(0,\ln(2)+1)$.

De lengte van het gevraagde lijnstuk is $\ln(2)+1-(\ln(2)-1)=\ln(2)+1-\ln(2)+1=2$.

G7b Stel $x_C = q$ dan is $x_D = 2q$

$g(q) = f(2q) = p \Rightarrow \ln(\frac{1}{q}) = \ln(8q)$ BV: $q > 0 \Rightarrow \frac{1}{q} = 8q \Rightarrow 8q^2 = 1 \Rightarrow q^2 = \frac{1}{8} \Rightarrow q = \sqrt{\frac{1}{8}}$ (voldoet) $\vee$ $q = -\sqrt{\frac{1}{8}}$ (vold. niet)

$p = g(q) = g(\sqrt{\frac{1}{8}}) = \ln(\frac{1}{\sqrt{\frac{1}{8}}}) = \ln(\frac{1}{\sqrt{\frac{1}{8}}} \cdot \frac{\sqrt{8}}{\sqrt{8}}) = \ln(\sqrt{8})$.

G8a $f(x) = x \cdot e^{-x+1} = 2x$

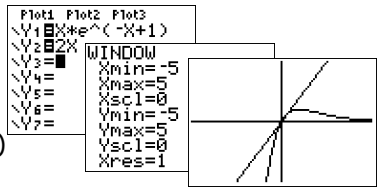
$x = 0 \vee e^{-x+1} = 2$

$x = 0 \vee -x + 1 = \ln(2)$

$x = 0 \vee -x = -1 + \ln(2)$

$x = 0 \vee x = 1 - \ln(2)$

$f(x) \le 2x$ (zie plot) voor $x \le 0 \vee x \ge 1 - \ln(2)$.



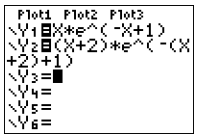
G8b $f(x) = x \cdot e^{-x+1} \Rightarrow f'(x) = 1 \cdot e^{-x+1} + x \cdot e^{-x+1} \cdot -1 = (1-x) \cdot e^{-x+1}$.

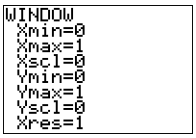
$f'(0) = (1-0) \cdot e^{-0+1} = 1 \cdot e^1 = e \Rightarrow y = ex$ is raaklijn in O

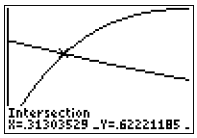
$x \cdot e^{-x+1} = ax$ heeft precies één oplossing (zie plot) $\Rightarrow a = e \vee a \le 0$.

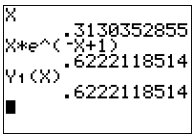
G8c $f(q) = f(q+2) = p \Rightarrow q \cdot e^{-q+1} = (q+2) \cdot e^{-(q+2)+1}$

intersect geeft $q \approx \ldots$ en $p = f(q) = f(q+2) \approx 0{,}62$.









G8d Stel $x_B = p$ dan is $x_C = e \cdot p$

$f(p) = f(ep) = q$

$p \cdot e^{-p+1} = ep \cdot e^{-ep+1}$ (intersect of)

$p = 0 \vee e^{-p+1} = e \cdot e^{-ep+1}$

$p = 0 \vee e^{-p+1} = e^{-ep+2}$

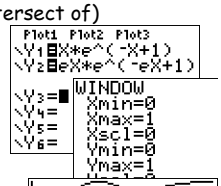
$p = 0 \vee -p + 1 = -ep + 2$

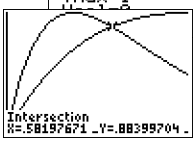
$p = 0 \vee ep - p = 1$

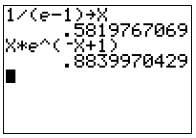
$p = 0 \vee (e-1) \cdot p = 1$

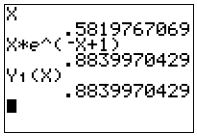
$p = 0$ (vold. niet) $\vee$ $p = \frac{1}{e-1}$

$q = f(p) = f(\frac{1}{e-1}) \approx 0{,}884$.









G9a $f(x) = g_4(x) \Rightarrow e^x - 3 = 4 \cdot e^{-x}$ $(\times e^x)$

$e^{2x} - 3e^x = 4$ (stel $e^x = t$)

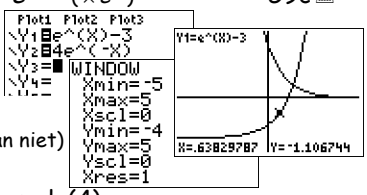
$t^2 - 3t - 4 = 0$

$(t-4) \cdot (t+1) = 0$

$t = e^x = 4 \vee t = e^x = -1$ (kan niet)

$x = \ln(4)$

$f(x) < g_4(x)$ (zie plot) voor $x < \ln(4)$.



G9b $f(x) = e^x - 3 \Rightarrow f'(x) = e^x$

$a = f'(x) = e^x = 2$

$x_{\text{raakp.}} = \ln(2) \Rightarrow$

$y_{\text{raakp.}} = f(\ln(2)) = e^{\ln(2)} - 3 = 2 - 3 = -1$

$-1 = 2 \cdot \ln(2) + b \Rightarrow b = -1 - 2 \cdot \ln(2)$.

G9c $g_p(x) = pe^{-x} \Rightarrow g_p'(x) = pe^{-x} \cdot -1 = -pe^{-x}$

$a = g_p'(x) = -pe^{-x} = 2$

$e^{-x} = -\frac{2}{p}$ BV: $p < 0$, want $e^{\cdots} > 0$

$-x = \ln(-\frac{2}{p})$

$x_{\text{raakp.}} = -\ln(-\frac{2}{p}) \Rightarrow$

$y_{\text{raakp.}} = g_p(-\ln(-\frac{2}{p})) = pe^{\ln(-\frac{2}{p})} = p \cdot -\frac{2}{p} = -2$

k: $y = 2x - 1 - \ln(2)$ (zie G9b) door raakpunt $\Rightarrow$

$-2 = 2 \cdot -\ln(-\frac{2}{p}) - 1 - 2\ln(2)$

$2 \cdot \ln(-\frac{2}{p}) = 1 - 2\ln(2)$

$\ln(-\frac{2}{p}) = \frac{1}{2} - \ln(2)$

$-\frac{2}{p} = e^{\frac{1}{2} - \ln(2)} = e^{\frac{1}{2}} \cdot e^{-\ln(2)} = \sqrt{e} \cdot e^{\ln(2^{-1})} = \sqrt{e} \cdot 2^{-1} = \frac{\sqrt{e}}{2}$

$p \cdot \sqrt{e} = -2 \cdot 2$

$p = -\frac{4}{\sqrt{e}}$ (voldoet).

G10a $f(x) = g_p(x) = 2$ BV: $x > 1$

$\ln(x-1) = p \cdot (x-1) = 2$ ✪.

$\ln(x-1) = 2$

$x - 1 = e^2$

$x = e^2 + 1$ (voldoet) (nu invullen in ✪)

$p \cdot (e^2 + 1 - 1) = 2$

$p \cdot e^2 = 2$

$p = \frac{2}{e^2}$.

G10b Snijden met de x-as ($y = 0$): $g_p(x) = 0 \Rightarrow p \cdot (x-1) = 0$

$x - 1 = 0$ (p is een of ander getal) $\Rightarrow x = 1 \Rightarrow A(1, 0)$.

Bekijk nu de grafiek van f en van de lijn g_p.

B is het midden van $AC \Rightarrow y_C = 2 \cdot y_B \Rightarrow$

$f(1+2a) = 2 \cdot f(1+a)$

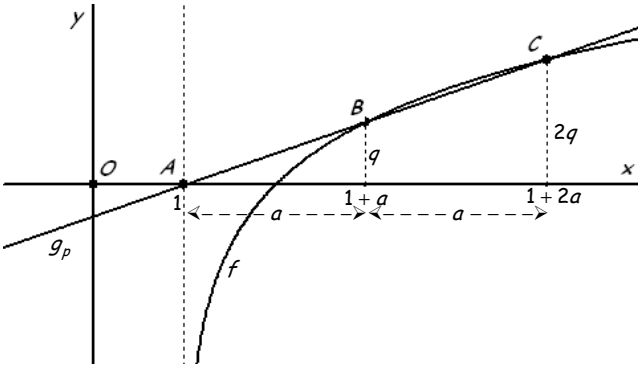
$\ln(1+2a-1) = 2 \cdot \ln(1+a-1)$

$\ln(2a) = 2 \cdot \ln(a)$ BV: $a > 0$

$\ln(2a) = \ln(a^2)$

$a^2 = 2a$

$a = 0$ (vold. niet) $\vee$ $a = 2$ (voldoet) ⇗



$a = 2 \Rightarrow x_B = 1 + a = 1 + 2 = 3$

$y_B = f(x_B) = \ln(3-1) = \ln(2)$ $\Rightarrow B(3, \ln(2))$.

B op $y = p \cdot (x-1) \Rightarrow \ln(2) = p \cdot (3-1) \Rightarrow 2p = \ln(2) \Rightarrow p = \frac{\ln(2)}{2}$.

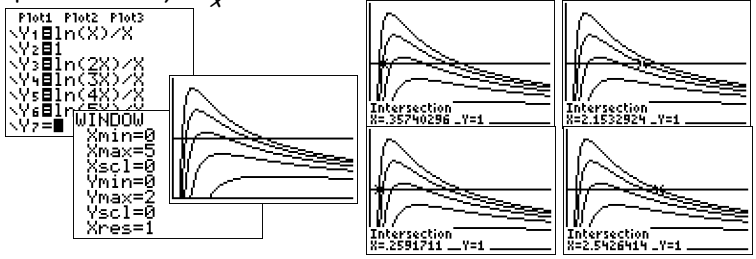
G11a $f_1(x)=\frac{\ln(x)}{x}$ BV: $x>0 \Rightarrow f_1'(x)=\frac{x\cdot\frac{1}{x}-\ln(x)\cdot 1}{x^2}=\frac{1-\ln(x)}{x^2}$.
$f_1'(x)=0$
$\frac{1-\ln(x)}{x^2}=0$ (teller = 0 $\Rightarrow$)
$\ln(x)=1$
$x_{top}=e^1=e \Rightarrow y_{top}=f_1(e)=\frac{\ln(e)}{e}=\frac{1}{e} \Rightarrow$ top $(e,\frac{1}{e})$.

G11b $f_k(x)=\frac{\ln(kx)}{x}$ BV: $kx>0 \Rightarrow f_k'(x)=\frac{x\cdot\frac{1}{x}-\ln(kx)\cdot 1}{x^2}=\frac{1-\ln(kx)}{x^2}$.
$f_k'(x)=0$
$\frac{1-\ln(kx)}{x^2}=0$ (teller = 0 $\Rightarrow$)
$\ln(kx)=1 \Rightarrow f_k(x)=\frac{1}{x}$. Dus de top ligt op de kromme $y=\frac{1}{x}$.

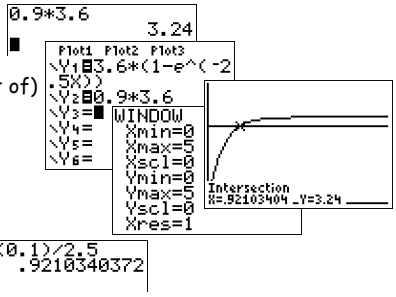
G11c $k\neq 0$ en k geheel $\Rightarrow k=1,\ 2,\ 3,\ \ldots$

k	y_1	y_2	x_A	x_B	AB
4	$\frac{\ln(4x)}{x}$	1	0,36	2,15	1,8
5	$\frac{\ln(5x)}{x}$	1	0,26	2,54	2,3

Dus vanaf $k=5$ is $AB>2$.

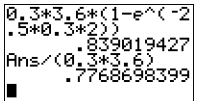


G12a 90% van 3,6 liter is 3,24 liter.
$L(t)=3{,}24$
$3{,}6\cdot(1-e^{-2{,}5t})=3{,}24$ (intersect of)
$1-e^{-2{,}5t}=0{,}9$
$-e^{-2{,}5t}=-0{,}1$
$e^{-2{,}5t}=0{,}1$
$-2{,}5t=\ln(0{,}1)$
$t=-\frac{\ln(0{,}1)}{2{,}5}\approx 0{,}9$ (seconden).

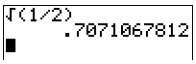


G12b De maximale hoeveelheid verse lucht is 2,2 liter (zie figuur G.2 in boek).
$\alpha\cdot 3{,}6=2{,}2$
$\alpha=\frac{2{,}2}{3{,}6}\approx 0{,}6$.

G12c $L_{0,3}(2)=0{,}3\cdot 3{,}6\cdot(1-e^{-2{,}5\cdot 0{,}3\cdot 2})\approx 0{,}84$ (liter).
Dus $\frac{0{,}84}{0{,}3\cdot 3{,}6}\times 100\%\approx 78\%$.



G12d $L_\alpha(t)=\alpha\cdot 3{,}6(1-e^{-2{,}5\alpha t})=3{,}6\alpha-3{,}6\alpha\cdot e^{-2{,}5\alpha t} \Rightarrow L_\alpha'(t)=-3{,}6\alpha\cdot e^{\boxed{-2{,}5\alpha t}}\cdot -2{,}5\alpha=9\alpha^2\cdot e^{-2{,}5\alpha t}$.
De maxmale vulsnelheid is $L_\alpha'(0)=9\alpha^2\cdot e^0=9\alpha^2$.
$9\alpha^2=4{,}5$
$\alpha^2=\frac{1}{2}$
$\alpha=\sqrt{\frac{1}{2}}\approx 0{,}71$.



Voor $g > 0$, $g \neq 1$, $a > 0$ en $b > 0$ geldt:

- $^g\log(a) + {}^g\log(b) = {}^g\log(a \cdot b)$
- $^g\log(a) - {}^g\log(b) = {}^g\log(\frac{a}{b})$
- $c \cdot {}^g\log(a) = {}^g\log(a^c)$
- $^g\log(a) = \frac{{}^p\log(a)}{{}^p\log(b)}$
- $g^{{}^g\log(a)} = a$
- $^g\log(g^c) = c$

Dus: $^g\log(1) = {}^g\log(g^0) = 0$
$^g\log(g) = {}^g\log(g^1) = 1$

$f(x) = e^x \Rightarrow f'(x) = e^x$
$f(x) = g^x \Rightarrow f'(x) = g^x \cdot \ln(g)$
$f(x) = \ln(x) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(ax) \Rightarrow f'(x) = \frac{1}{x}$
$f(x) = \ln(x^a) \Rightarrow f'(x) = \frac{a}{x}$
$f(x) = {}^g\log(x) \Rightarrow f'(x) = \frac{1}{x \cdot \ln(g)}$